TOPIC 1: <u>ALGEBRA</u>

Definition: Algebra is a type of Mathematics in which letters and symbols are used to represent quantities.

Examples: 1) James has x kg of rice.

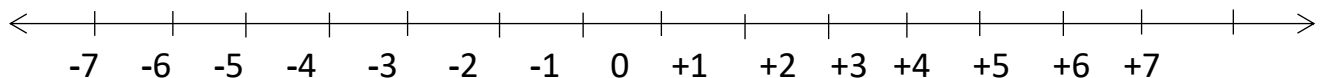→ Then, in the sentence above, x represents the number of kg of rice James has.

2) 3y – 5 → where y represents the unknown value.

1. <u>INTEGERS</u>

Definition: Integers are positive and negative numbers together with zero plotted in equal distances on number line.

A. <u>Integers on a number line</u>

Study the number line below.

A set of integers include:

1) Positive integers (1, 2, 3, 4, 5, 6, ………….)
2) Zero (0)
3) Negative numbers (-1, -2, -3, -4, -5, -6, ………)

Notice: a) Zero is neither negative nor positive integer.

b) Any integer without a sign is a positive integer.

→ +6 is the same as 6 as 10 is the same as +10.

B. <u>Explaining integers</u>

- If I have no money at all, it means 0 money.
- If I have been given some 100 F, it means +100 F.
- If I have lost 2,000 F, it means -2,000 F.
- If a team scores 2 goals, it means +2.
- If a trader makes a profit of 5,000 F, it means +5,000 F.

- If a shopkeeper makes a loss of 3,000 F, it means -3,000 F.

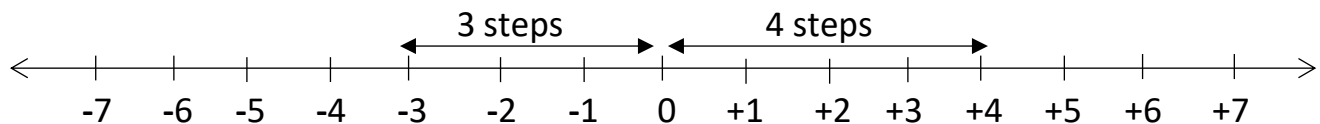# C. <u>Distance between two integers</u>

<u>i) Two integers on the same side of zero</u>

If two integers are on the same side of zero, the distance between them is the difference of their magnitude.

Magnitude of a number, is the distance from 0 to that number.

Study the number below and state the magnitude of: a) -3

b) +4

Solution:

a) The magnitude of -3 = 3

b) The magnitude of +4 = 4

<u>Examples</u>

1) What is the distance between + and +10?
   - **Magnitude of +2 = 2**
   - **Magnitude of +0 = 10**
   - **Difference=10 – 2 = 8 steps**
2) Find the distance between -6 and -13.
   - **Magnitude of -6 = 6**
   - **Magnitude of -13 = 13**

<u>ii) Two integers on opposite sides of zero</u>

If two numbers are on opposite sides of zero, the distance between them is the sum of their magnitudes.

<u>Examples</u>

1) What is the distance between -5 and +4?
   - **Magnitude of -6 = 6**
   - **Magnitude of +4 = 4**
   - **Sum = 6 + 4 = 10 steps**
2) Find the distance between -10 and +10.
   - **Magnitude of -10 = 10**

- **Magnitude of +10 = 10**
- **Sum = 10 + 10 = 20**
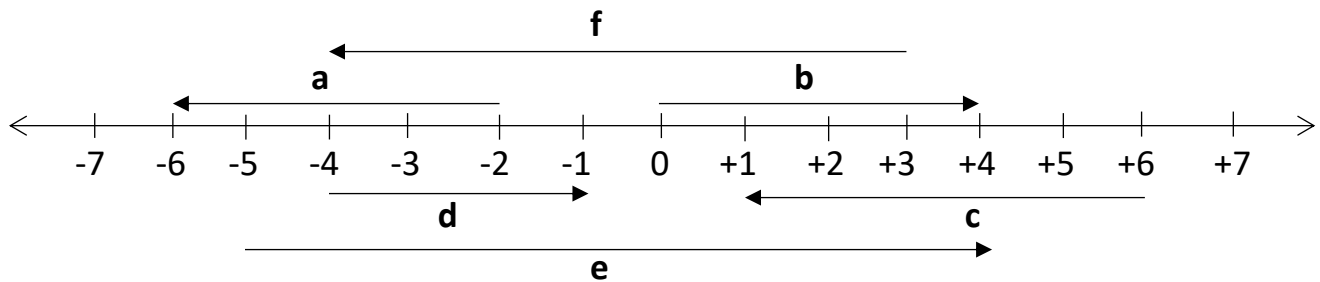
D. <u>Arrows on the number line</u>

Note: * If the arrow looks at the left, the answer is a negative.

* If the arrow looks at the right, the answer is a positive.

* To find the length of the arrow, just find the distance between two numbers.

<u>Examples</u>

1) What integers are represented by the arrow?

<u>Solution</u>

- Magnitude of -2 = 2
- Magnitude of -6 = 6
- Difference = 6 − 4 = 2
  Arrow a indicates -3

- Magnitude **of 0 = 0**
- **Magnitude of +4 = 4**
- **Difference = 4 − 0 = 4**
  Arrow b indicates +4

- Magnitude of +1 = 1
- Magnitude of +6 = 6
- Difference = 6 − 1 = 5
  Arrow c indicates -5

- Magnitude of -1 = 1
- Magnitude of -4 = 4
- Difference = 4 − 1 = 3
  Arrow d indicates =+3

- Magnitude of -5 = 5
- Magnitude of +4 = 4
- Sum = 4 + 5 = 9
  Arrow e indicates +9

- Magnitude of -4 = 4
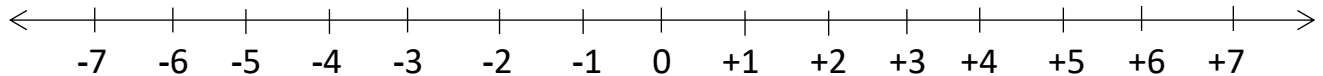- Magnitude of +3 = 3
- Sum = 4 + 3 = 7
  Arrow f indicates -7

2) How far is it from -9 to -7?

- Magnitude of -9 = 9

- Magnitude of -7 = 7
- Difference = 9 – 2 = 2
  It is +2

E. Comparing of integers

Study the number line below.

NOTE:

1) On a number line, the integers are always in order from the smallest to the biggest.
2) Any integer on the right is greater than any integer on its left on a number line.
3) Any integer on the left is smaller than any integer on its right on a number line.
4) All positive integers are greater than all negative integers.
5) Zero "0" is greater than all negative integers.

Examples.

Compare the following integers:

a) + 2 > -9

b) -100 < +6

c) +80 > -5,000

d) -10 > -30

e) -80 <   0

f) 1 < -9

F. Ordering integers

Descending order (decreasing order) means to arrange from the biggest number to the smallest number.

Ascending order (increasing order) means to arrange from the smallest number to the biggest number.

Examples

1) Arrange the following numbers in descending order.

-6 , +1 , -12 , -7 , +3 , -15 , 0 , -4

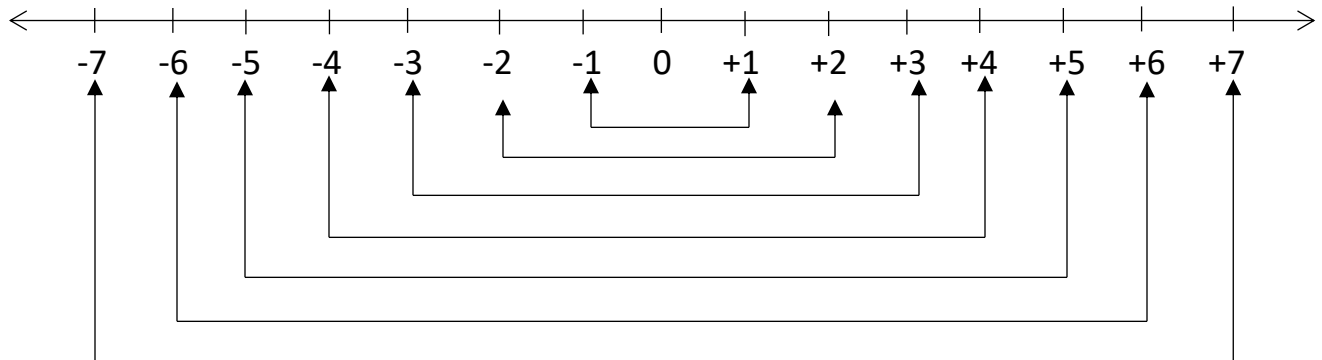**Answer: +3 , +1 , 0 , -4 , -6 , -7 , -12 , -15**

2) Re-arrange the numbers below in increasing order.

7 , -5 , -10 , 0, -3 , -9 , 6

Answer: -10 , -9 , -5 , -3 , 0 , 6 , 7

G. <u>Opposites or inverses</u>

Study the number line below.

➢ The opposite of -1 = +1
➢ The opposite of -2 = +2
➢ The opposite of +3 = -3
➢ The opposite of -4 = +4
➢ The opposite of +5 = -5
➢ The opposite of -6 = +6
➢ The opposite of +7 = -7

<u>NOTE:</u> When two numbers have the same magnitudes, and the two numbers are on opposites of zero, one number is the **opposite** of another.

<u>Additive inverses</u>

Any integer added to its opposite the result is zero.

<u>Examples</u>

1) +3 + -3 = 0

2) +7 - 7 = 0

3) (-13) + (+13) = 0

4) -8 + 8 = 0

Then, * +3 is an additive inverse of -3

   * -30 is an additive inverse of +30

   * +100 is an additive inverse of -100

   * -80 is an additive inverse of +80

G. Operations of integers

1. Addition and subtraction

- When adding or subtracting two integers that have the same signs, simply add them and keep their sign.

Examples

Work out: $+6 +3 = +9$

    $-4 - 2 = -6$

    $+8 + 2 = +10$

    $-5 - 6 = -11$

- When adding or subtracting two integers that have different signs, simply subtract them and write the sign of a big number.

Examples

Work out: $+5 - 2 = +3$

    $-6 + 3 = -3$
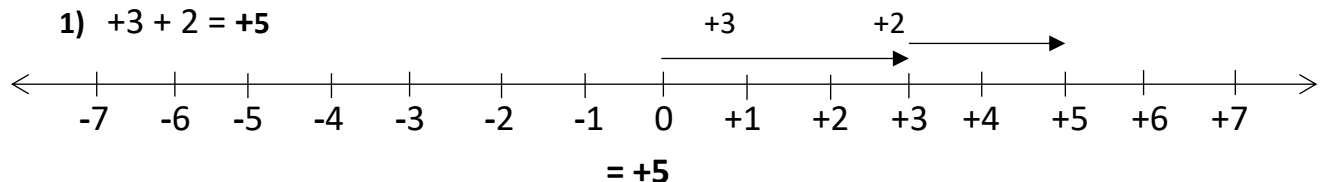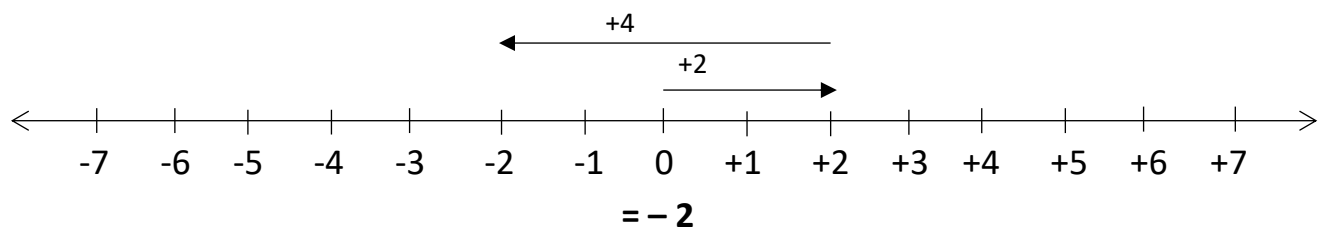
    $+9 - 14 = -5$

    $-3 + 7 = +4$

Adding and subtracting integers using a number line
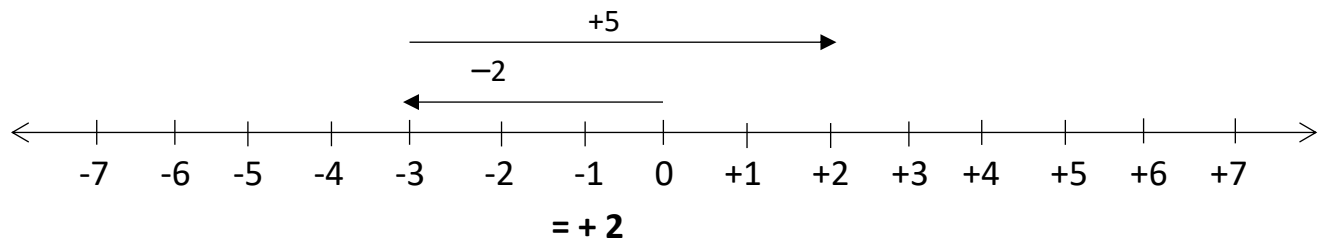
Work out the following using a number line.

1) $+3 + 2 = $ **+5**

$= $ **+5**

2) $+2 - 4 = $ **-2**

$= -$ **2**

3) $-3 + 5 = $ **+2**

= + 2

More about addition and subtraction.

Work out the following:

    a) −2 + 4 − 6 **= −2 − 6 + 4**

                = − 8 + 4

                = −4

    b) (−4) + (+7) − (+10) + (+5) + (−12) **= −4 + 7 −10 + 5 − 12**

                                  = −4 − 10 − 12 + 7 + 5

                                    = −26 + 12

                                    = −14

    c) (+3) − (+6) + (−4) − (−7) + (+4) + (−3) + (+9) **= +3 − 6 − 4 + 7 + 4 − 3 + 9**

                                    = + 3 + 7 + 9 − 6 − 4 − 3

                                    = + 19 − 13

                                    = + 6

2. Multiplication and division.

- The product or quotient of the same signs is always **positive.**

    Therefore, positive × positive = positive $+ \times + = +$

              negative × negative = positive $- \times - = +$

              positive ÷ positive = positive $+ \div + = +$

              negative × negative = positive $- \times - = +$

                Examples.

    Effectuate: + 3 × + 5 = + 20

              − 4 ÷ − 2 = +2

              + 3 × + 2 = + 6

              −6 ÷ − 3 = +2

- The product or quotient of different signs is always **negative.**

    Therefore, negative × positive = negative $- \times + = -$

              positive × negative = negative $+ \times - = -$

$$negative \div positive = negative - \div + = -$$
$$positive \div negative = negative + \div - = -$$

Examples.

Calculate: $+ 3 \times -5 = -15$

$$-8 \div + 2 = -4$$
$$-3 \times 2 = -6$$
$$+10 \div 2 = -5$$

Exponents of integers.

Exponent is a raised figure that shows how many times a number must be multiplied by itself.

Example: $x^4 = x \times x \times x \times x$

Calculate the following:

1) $(-2)^2 = -\mathbf{2} \times (-\mathbf{2})$
$$= +\mathbf{4}$$

2) $(-4)^3 = -\mathbf{4} \times (-\mathbf{4}) \times (-\mathbf{4})$
$$= +\mathbf{16} \times (-\mathbf{4})$$
$$= -\mathbf{64}$$

3) $(-3)^3 + (+2)^4 = -\mathbf{3} \times (-\mathbf{3}) \times (-\mathbf{3}) + (+\mathbf{2}) \times (+\mathbf{2}) \times (+\mathbf{2}) \times (+\mathbf{2})$
$$= +\mathbf{9} \times (-\mathbf{3}) + (+\mathbf{4}) \times (+\mathbf{4})$$
$$= -\mathbf{27} + (+\mathbf{16})$$
$$= -\mathbf{27} + \mathbf{16}$$
$$= -\mathbf{11}$$

4) $\dfrac{(-5)^2 - (+3)^2}{(-2)^3} = \dfrac{-\mathbf{5} \times (-\mathbf{5}) - (+\mathbf{3}) \times (+\mathbf{3})}{-\mathbf{2} \times (-\mathbf{2}) \times (-\mathbf{2})}$

$$= \dfrac{+\mathbf{25} - (+\mathbf{9})}{+\mathbf{4} \times (-\mathbf{2})}$$

$$= \dfrac{+\mathbf{25} - \mathbf{9}}{-\mathbf{8}}$$

$$= \dfrac{+\mathbf{16}}{-\mathbf{8}}$$

$$= -\mathbf{2}$$

5) $\dfrac{(-4)^2 - (-2)^4}{-3^2} = \dfrac{-\mathbf{4} \times (-\mathbf{4}) - (-\mathbf{2}) \times (-\mathbf{2}) \times (-\mathbf{2}) \times (-\mathbf{2})}{-\mathbf{3} \times \mathbf{3}}$

$$= \dfrac{+\mathbf{16} - (+\mathbf{4}) \times (-\mathbf{2}) \times (-\mathbf{2})}{-\mathbf{9}}$$

$$= \dfrac{+\mathbf{16} - (-\mathbf{8}) \times (-\mathbf{2})}{-\mathbf{9}}$$

$$= \dfrac{+\mathbf{16} - (+\mathbf{16})}{-\mathbf{9}}$$

$$=\frac{+16-16}{-9}$$

$$=\frac{0}{-9}$$

$$= 0$$

6) $\frac{-3^3-2^3}{-5 \times 7} = \frac{-3 \times 3 \times 3-2 \times 2 \times 2}{-35}$

$$=\frac{-27-8}{-35}$$

$$=\frac{-35}{-35} = +1$$

H. <u>Substitution.</u>

In Algebra, substitution mean replacing letters with numbers.

<u>Examples</u>.

1) If a=1, b=-1, find the value of ab – a

$$\boldsymbol{ab} - \boldsymbol{a}= \mathbf{1 \times (-1) - 1}$$

$$=\mathbf{-1 - 1}$$

$$= \textbf{-2}$$

2) Given that a=-1, b=2 and c=1 find the value of ba – ca

ba – ca = 2 × (–1) – 1 × (–1)

$$= \textbf{- 2 + 1}$$

$$= \textbf{-1}$$

3) If x=-2, y=-1 ans z=2, find the value of xyz – xy

xyz – xy = -2 × (-1) × 2 – (-2) × (-1)

$$= \textbf{+2 × 2 – (+2)}$$

$$=\textbf{+4 – 2}$$

$$= \textbf{+2}$$

4) Given that n=-4, p=3 and b=-5 find the value of bp – npb

bp – npb = -5 × 3 – (-4) × 3 × (-5)

$$= \textbf{-15 – (-12) × (-5)}$$

$$= \textbf{-15 – (+60)}$$

$$= \textbf{-15 – 60}$$

$$= \textbf{-75}$$

5) If n=-2, m=-1 and p=2, find the value of $\frac{nm-mp}{np}$

$$\boldsymbol{\frac{nm-mp}{np}} =\frac{\textbf{-2} \times \textbf{(-1)- (-1)} \times \textbf{2}}{\textbf{-2} \times \textbf{2}}$$

$$= \frac{\textbf{+2-(-2)}}{\textbf{-4}}$$

$$= \frac{+2 + 2)}{-4}$$

$$= \frac{+4}{-4}$$

$$= -1$$

6) Find the value of $\frac{a^2 + b^3}{a-b}$ if a=-3, b=-2

$$\frac{a^2 + b^3}{a-b} = \frac{-3 \times (-3) + (-2) \times (-2) \times (-2)}{-3 - (-2)}$$

$$= \frac{+9 + (+4) \times (-2)}{-3 + 2} = \frac{+9 + (-8)}{-1} = \frac{+9 - 8}{-1} = \frac{+1}{-1} = \textbf{-1}$$

II. ALGEBRAIC EXPRESSIONS.

A. Like terms and unlike terms.

Compare the like and unlike terms.

Like terms	Unlike terms
1) 3x and x	**1)** x and y
2) 2y and 10y	**2)** 2y and 3x
3) ab and 2ab	**3)** ab and ab^2
4) 5p and 8p	**4)** p and q
5) n^2 and $3n^2$	**5)** 2m and 7b
6) $4y^3$ and $2y^3$	**6)** n^2 and 2n

B. Writing phrases for algebraic expressions.

1) Add b to a = **a+b**

2) Subtract b from a **= a–b**

3) Multiply b by a = **ab**

4) Divide b by a = $\frac{b}{a}$

5) Add 5 to n = **n + 5**

6) Subtract 5 from n = **n – 5**

7) Subtract x from 2 = 2 – x

8) Multiply n by 5 = **5n**

9) Divide 5 b n = $\frac{5}{n}$

10) 2 more than x = **x+2**

11) 2 less than x = **x – 2**

12) Twice y = **2y**

or Double y

or two times y

13) Three times p = **3p**

or Thrice p

14) Half a = $\frac{a}{a2}$

15) 5 years younger than x = **x-5**

16) 5 years older than y = **y+7**

17) Twice as old as n = **2n**

18) Average of a and b = $\frac{a+b}{2}$

19) Square p **= p²**

20) Multiply the square of a by 3 = **3a²**

21) Sum of 8 and x **= 8+x**

22) Triple a = 3a

23) Square root of a =$\sqrt{a}$

24) Increase x by 4 = **x+4**

25) Decease 6 by m = **5-m**

26) Thrice the difference between a and b = **3(a-b)**

27) Five times the sum of x and y = **5(x+y)**

28) Thrice n added 6 = **3n+6**

29) Square the sum of a and 4 **= (a+4)²**

30) A third of a p = $\frac{1}{3}$**p or** $\frac{p}{3}$

31) Twice m subtracted 4 = **2m-4**

32) Divide x by 3 and add 5 = $\frac{x}{3}$**+ 5**

33) Subtract the sum of y and 2 from the difference between a and 4 **=(a-4)-(y+2)**

34) Thrice n subtracted from 6 = **6 – 3n**

C. <u>Collecting like terms</u>.

Like terms have exactly the same letters (unknown factors).

<u>Examples</u>.

Simplify completely:

1) r + r + r + r = **4r**

2) x + y + x + y + x + y + x = **x + x + x + x + y + y + y**

$$=4\ x + 3y$$

3) 2 x + 3y - x – 4y + 3 x + y - 4 x – 2y= **2 x - x + 3 x - 4 x + 3y – 4y + y – 2y**

$$= 2\ x + 3\ x - x - 4\ x + 3y + y – 4y – 2y$$
$$= 5\ x - 5\ x + 4y – 6y$$
$$= -2y$$

4) 2a – 3b + 4 – 4a +2b – 7 +3a – 2 – 3b =**2a – 4a + 3a – 3b + 2b – 3b + 4 – 7 – 2**

$$= 2a + 3a – 4a– 3b – 3b + 2b + 4 – 9$$
$$= 5a – 4a – 6b + 2b – 5$$
$$= a – 4b – 5$$

5) m + 2n – 2 – 2m + 3 – 3n + 4m – 5n – 6 – 3m + 3n + 2

$$= m – 2m + 4m – 3m + 2n – 3n – 5n + 3n – 2 + 3 – 6 + 2$$
$$=m + 4m – 2m – 3m + 2n + 3n – 3n – 5n – 2 – 6 + 3 + 2$$
$$=5m – 5m + 5n – 7n – 8 + 5$$
$$= -2n – 3$$

d. <u>Simplification involving brackets</u>.

1) 2(3x + 2) – 3(x +4) = **6 x + 4 - 3 x – 12**

$$=6\ x - 3\ x + 4 – 12$$

$$= 3x - 8$$

2) $4(x + 2y - 3) - 2(2x - 3y) + 4$ = **4 x + 8y – 12 - 4 x + 6y + 4**

$$=4 x - 4 x + 8y + 6y - 12 + 4$$

$$=14y - 8$$

3) -2(-2m + p) – 3(p + 3m + 5) = **+4m – 2p – 3p – 9m – 15**

$$= +4m - 9m - 2p - 3p - 15$$

$$= - 5m - 5p - 15$$

4) $3 - (b + y - 1) - (b + 2y - 1)$ = **3 – b – y + 1 – b – 2y + 1**

$$= -b - b - y - 2y + 3 + 1 + 1$$

$$= -2b - 3y + 5$$

e. **More about simplification**.

1) Add $x - 4$ to $x + 1$

$$= x + 1 + x - 4$$

$$= x + x + 1 - 4$$

$$= 2x - 3$$

2) Subtract y + 1 from 2y + 3

$$= 2y + 3 - (y + 1)$$

$$= 2y - y + 3 - 1$$

$$= y + 2$$

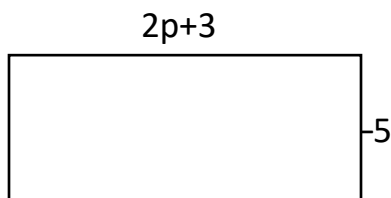**3)** Calculate the perimeter of the figure below.
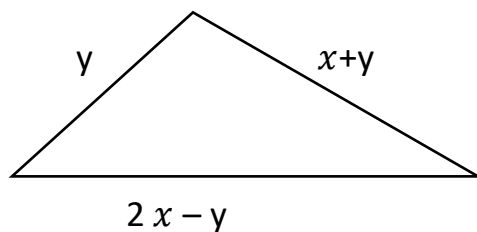
P= **sum of all sides**

$$= y + x + y + 2x - y$$

$$= y + y - y + x + 2x$$

$$= 2y - y + 3x$$

$$= y + 3x$$

4) Find the perimeter of this rectangle.

$$P=(L+W) \times 2$$
$$= (2p+3+p-5) \times 2$$
$$=4p + 6 + 2p - 10$$
$$=4p + 2p - 10 + 6$$
$$=6p - 4$$

5) Subtract 3m − 1 from 5m − 3
$$= 5m - 3 - (3m - 1)$$
$$= 5m - 3 - 3m + 1$$
$$= 5m - 3m - 3 + 1$$
$$= 2m - 2$$

f. Fractional terms

A fractional term is a number with a numerator and denominator.

Examples.

Simplify completely:

a) $\frac{x}{2} + \frac{x}{3}$

$$\frac{x}{2} + \frac{x}{3} = \frac{3x+2x}{6}$$
$$= \frac{5x}{6}$$

b) $\frac{m}{2} - \frac{m}{5}$

$$\frac{m}{2} - \frac{m}{5} = \frac{5m-2m}{10}$$
$$= \frac{3m}{10}$$

c) $y + \frac{y}{3}$

$$y + \frac{y}{3} = \frac{3y+y}{3}$$
$$= \frac{4y}{3}$$

d) $\frac{p}{2} + \frac{p}{3} + \frac{p}{3}$

$$\frac{p}{2} + \frac{p}{3} + \frac{p}{3} = \frac{3p+2p+2p}{6}$$
$$= \frac{7p}{6}$$

e) $\frac{x-1}{2} + \frac{x+1}{3}$

$$\frac{x-1}{2} + \frac{x+1}{3} = \frac{3(x-1)+2(x+1)}{6}$$
$$= \frac{3x-3+2x+2}{6}$$

$$= \frac{3x+2x-3+2}{6}$$
$$= \frac{5x-1}{6}$$

f) $\frac{2n+3}{4} - \frac{n-5}{3}$

$$\frac{2n+3}{4} - \frac{n-5}{3} = \frac{3(2n+3)-4(n-5)}{12}$$
$$= \frac{6n+9-4n+20}{12}$$
$$= \frac{6n-4n+9+20}{12}$$
$$= \frac{2n+29}{12}$$

EXERCISES

Simplify completely:

1) $\frac{x+2}{5} - \frac{x-5}{2}$

2) $\frac{x+3}{9} - \frac{x+2}{3}$

3) $\frac{2y+1}{7} + \frac{3y-3}{2}$

4) $\frac{m}{4} - \frac{m}{3}$

5) $\frac{x+y-1}{2} - \frac{x-y-2}{3}$

6) $\frac{a}{8} + \frac{a}{6}$

7) $p + \frac{2p}{3} + \frac{p}{5}$

8) $5m + \frac{xm}{3}$

9) $\frac{x-1}{3} + \frac{x+1}{4}$

10) $\frac{x}{3} + \frac{x}{3}$

11) $\frac{2b+3}{5} + \frac{b-2}{3}$

12) $\frac{k}{2} - 2k$

13) $y - \frac{y-2}{2}$

Removing brackets involving brackets

Simplify completely:

a) $\frac{1}{2}$ (8a + 4b)

$\frac{1}{2}$ **(8a + 4b) =** $\frac{1}{2} \times 8a + \frac{1}{2} \times 4b$

$\qquad$ **= 4a + 2b**

b) $\frac{1}{4} (4x + 12)$

$\frac{1}{4} (4x + 12) = \frac{1}{4} \times 4x + \frac{1}{4} \times 12$

$\qquad$ **= x + 3**

c) $\frac{3}{4}(8m - 12p + 16)$

$\frac{3}{4}(8m - 12p + 16) = \frac{3}{4} \times 8m - \frac{3}{4} \times 12p + \frac{3}{4} \times 16$

$\qquad$ =6m − 9p + 12

d) $\frac{1}{2}(2x + 8) + \frac{1}{3}(6x + 9)$

$\frac{1}{2}(2x + 8) + \frac{1}{3}(6x + 9) = \left(\frac{1}{2} \times 2x + \frac{1}{2} \times 8\right) + (\frac{1}{3} \times 6x + \frac{1}{3} \times 9)$

$\qquad$ =x + 4 + 2x + 3

$\qquad$ = x + 2x + 4 + 3

$\qquad$ = 3x + 7

Exercises.

Simplify:

1) $\frac{1}{3}(6a - 9) + \frac{2}{3}(15x + 18)$

2) $\frac{1}{10}(30m + 20) + \frac{1}{7}(14m - 21)$

3) $\frac{2}{3}(6x - 12y + 9) - \frac{3}{4}(4x + 4y - 12)$

4) $\frac{1}{6}(6x - 18y - 12) + \frac{1}{3}(3x - 6y + 150$

5) $\frac{1}{5}(10p - 5b) - \frac{2}{3}(6p - 3)$

g. <u>Equations.</u>

An equation is a statement showing that two amounts or values are equal.

<u>Examples</u>.

   a) 2x = 6

   b) 2y + 4 = 9

   c) 3p – 2 = 2p + 6

   d) $\frac{x+1}{4} = \frac{x-1}{3}$

   e) 2(m+1) = 3(m-1)

<u>Solving equations</u>.

<u>Note</u>: 1) Solving equations means finding the value of the unknown factor.

    2) When solving equations, collect like terms.

    3) When a number crosses the equal sign, it changes its sign.

1. <u>Equations with one unknown factor on one side.</u>

Solve:

a) $x + 6 = 4$

$x + 6 = 4$

$x = 4 - 6$

$x = -2$

b) $y - 3 = 2$

$y - 3 = 2$

$y = 2+3$

$y = 5$

c) $2m = 8$

$2m = 8$

$\frac{2m}{2} = \frac{8}{2}$

$m = 4$

d) $2b + 3 = 5$

$2b + 3 = 5$

$2b = 5 - 3$

$2b = 2$

$\frac{2b}{2} = \frac{2}{2}$

$b=1$

e) $3m - 4 = 5$

$3m - 4 = 5$

$3m = 5 + 4$

$3m = 9$

$\frac{3m}{3} = \frac{9}{3}$

$m=$

2. Equations with unknown factors on both sides.

Solve:

a) $4x + 5 = x + 14$

$4x + 5 = x + 14$

$4x - x = 14 - 5$

$3x = 9$

$\frac{3x}{3} = \frac{9}{3}$

$x=3$

b) $3m - 6 = m + 8$

$3m - 6 = m + 8$

$3m - m = 8 + 6$

$2m = 14$

$\frac{2m}{2} = \frac{14}{2}$

$m = 7$

c) $6y - 5 = 10y - 13$

$6y - 5 = 10y - 13$

$6y - 10y = -13 + 5$

$-4y = -8$

$4y = 8$

$\frac{4y}{4} = \frac{8}{4}$

$y = 2$

d) $2p + 6 = p + 11$

$2p + 6 = p + 11$

$2p - p = +11 - 6$

$P = 5$

e) $4t - 7 = 5t + 1$

$4t - 7 = 5t + 1$

$4t - 5t = 1 + 7$

$-t = 7$

$t = -7$

f) $3n + 4 = 5n - 2$

$3n + 4 = 5n - 2$

3n − 5n = - 2 − 4   **n = 3**

-2n = -6

$$\frac{-2n}{-2} = \frac{-6}{-2}$$

Exercises.

Solve:

1) 2x + 8 = 4x + 6
2) -4y + 5 = y − 5
3) 2+ 3p = 6p − 4
4) 5y + 2 = 2y = 6
5) m + 3 = 2 − m
6) 2b − 2 = b + 4
7) x − 1 = 2x + 3
8) 2y + 5 = 3y − 2
9) 2t − 5 = 4t + 7
10) 3y + 3 = 5y − 1

3. Equations involving brackets.

Solve:

a) 3(x + 5) = 2(x + 8)

3(x + 5) = 2(x + 8)

3x + 15 = 2x + 16

3x − 2x = 16 − 15

x = 1

3n − 3 = −2n − 1

3n + 2n = − 1 + 3

5n = + 2

$$\frac{5n}{5} = \frac{2}{5}$$

$$n = \frac{2}{5}$$

b) 4(y − 3) = 2(y + 8)

4(y − 3) = 2(y + 8)

4y − 12 = 2y + 16

4y − 2y = 16 + 12

2y = 28

$$\frac{2y}{2} = \frac{28}{2}$$

y = 14

d) 2(2p − 4) = −3(−p − 3)

2(2p − 4) = −3(−p − 3)

4p − 8 = +3p + 9

4p − 3p = 9 + 8

P = 17

e) 5(y+ 1) −3(y − 1) = 14

5(y+ 1) −3(y − 1) = 14

5y + 5 − 3y + 3 = 14

c) 3(n − 1) = −(2n + 1)

3(n − 1) = −(2n + 1)

5y − 3y = 14 − 5 − 3

2y = 14 − 8

2y =6

$\frac{2y}{2} = \frac{6}{2}$

y = 3

f) 4(x − 1) − 3(x − 2) + 2 = 0

4(x − 1) − 3(x − 2) + 2 = 0

4x − 4 − 3x + 2 + 2 = 0

4x − 3x = 0 − 2 − 2

x = - 4

Exercises.

Solve:

1) 2(y − 3) + 4 = - (2y + 2)

2) 2(3x − 1) − 4(x − 1) = 4

3) 3(2y + 1) − 2 (y + 4) = 35

4) 4(p − 1) − 3(p + 2) = 0

5) 6(m + 1) − 13 = m + 2

4. Equations involving fractions.

a. Equations with one fraction on each side.

Rule: Cross multiply. Note that, when $\frac{a}{b} = \frac{c}{d}$ then, $a \times d = b \times c$

Examples.

Solve:

a) $\frac{3x+1}{2} = \frac{2x+14}{3}$

$\frac{3x+1}{2} = \frac{2x+14}{3}$

$3(3x + 1) = 2(2x + 14)$

$9x + 3 = 4x + 28$

$9x − 4x = 28 − 3$

$5x = 25$

$\frac{5x}{5} = \frac{25}{5}$

$x = 5$

b) $\frac{4n+7}{3} = \frac{6n+8}{4}$

$\frac{4n+7}{3} = \frac{6n+8}{4}$

4(4n+7) = 3(6n+8)

16n + 28 = 18n + 24

16n − 18n = 24 − 28

-2n = -4

$\frac{-2n}{-2} = \frac{-4}{-2}$

n = 2

c) $\frac{4y-6}{5} = \frac{3y-15}{2}$

$$\frac{4y-6}{5} = \frac{3y-15}{2}$$

2(4y − 6) = 5(3y − 15)

8y − 12 = 15y − 75

8y − 15y = - 75 + 12

- 7y = -63

$$\frac{-7y}{-7} = \frac{-63}{-7}$$

y = 9

d) $\frac{x+1}{2} = \frac{x-1}{3}$

$$\frac{x+1}{2} = \frac{x-1}{3}$$

3(x + 1) = 2(x − 1)

3x + 3 =2x − 2

3x − 2x = -2 − 3

x= - 5

Exercises.

Solve:

1) $\frac{m+6}{3} = \frac{m+4}{9}$

5) $\frac{x+3}{3} = \frac{5x+1}{9}$

9) $\frac{13y-5}{9} = \frac{11y-8}{7}$

2) $\frac{2p-10}{5} = \frac{p-4}{2}$

6) $\frac{x+7}{6} = \frac{x+8}{3}$

10) $\frac{t-2}{7} = \frac{4-8t}{28}$

3) $\frac{3x+1}{4} = \frac{x+2}{2}$

7) $\frac{2m-4}{5} = \frac{6m+2}{20}$

11) $\frac{y+1}{5} = \frac{y-1}{6}$

4) $\frac{3y-1}{2} = \frac{7y+1}{6}$

8) $\frac{3x}{4} = \frac{16x-1}{20}$

12) $\frac{8x-18}{6} = \frac{6x+10}{14}$

b. Equations with more than on fraction.

Rule: Put them on the common denominator and then cancel the denominators.

Examples.

Solve:

a) $\frac{t}{5} + t = 60$

$\frac{t}{5} + t = 60$

$\frac{t+5t}{5} = \frac{300}{5}$

t+5t = 300

6t = 300

$\frac{6t}{6} = \frac{300}{6}$

t = 50

$\frac{x+1}{4} - 7 = x$

$\frac{x+1-28}{4} = \frac{4x}{4}$

$x + 1 - 28 = 4x$

$x - 4x = -1 + 28$

$-3x = +27$

$\frac{-3x}{-3} = \frac{+27}{-3}$

$x = -9$

y - $\frac{y-1}{3} = 7$

$\frac{3y-(y-1)}{3} = \frac{7}{3}$

3y − y +1 = 7

3y − y = 7 − 1

2y = 6

$\frac{2y}{2} = \frac{6}{2}$

y=3

b) $\frac{x+1}{4} - 7 = x$

c) y - $\frac{y-1}{3}$

d) y - $\frac{y-1}{3} = 7$

e) $\frac{m+1}{3} + \frac{m}{4} = 2$

$$\frac{m+1}{3} + \frac{m}{4} = 2$$

$$\frac{4(m+1)+3m}{12} = \frac{24}{12}$$

4(m+1) + 3m = 24

4m + 4 + 3m = 24

4m + 3m = 24 − 4

7m = 20

$$\frac{7m}{7} = \frac{20}{7}$$

$$m=\frac{20}{7}$$

$$m=4\frac{2}{7}$$

f) $\frac{p+7}{6} = \frac{p+8}{3} - 4$

$$\frac{p+7}{6} = \frac{p+8}{3} - 4$$

$$\frac{p+7}{6} = \frac{2(p+8)-24}{6}$$

p+7 = 2(p+8) − 24

p+7 = 2p + 16 − 24

p − 2p = +16 − 24 − 7

-p = +16 − 31

-p = -15

P = 15

g) $\frac{y}{2} + \frac{1}{3} = \frac{y}{4} - \frac{1}{6}$

$$\frac{y}{2} + \frac{1}{3} = \frac{y}{4} - \frac{1}{6}$$

$$\frac{6y+4}{12} = \frac{3y-2}{12}$$

6y + 4 = 3y − 2

6y − 3y = − 2 − 4

3y = - 6

$$\frac{3y}{3} = \frac{-6}{3}$$

y = -2

Exercises.

Solve:

1) $\frac{2x}{4} - \frac{x+1}{3} = 2$

2) $\frac{2x-3}{2} - \frac{x+1}{5} = \frac{7}{10}$

3) $\frac{x}{6} + \frac{1}{2} = \frac{x}{3} - \frac{1}{3}$

4) $\frac{2x}{3} + \frac{1}{2} = \frac{x}{2} + \frac{3}{5}$

5) $\frac{2y+1}{3} + 2 = 3y$

6) $4m + 7 - \frac{4m}{7} = 31$

7) $\frac{x}{5} + 7 = 11$

8) $\frac{2}{3}p - p = 5$

9) $\frac{x+6}{8} + \frac{x}{4} = 3$

10) $\frac{y-2}{2} - \frac{y+1}{3} = 2$

11) $\frac{x}{2} + \frac{1}{4} = \frac{2x}{5}$

12) $\frac{y+2}{3} + \frac{2}{5} = 1$

5. Equations involving square roots.

Solve:

a) $y^2 = 4$

$y^2 = 4$

$y = \sqrt{4}$

y =2

b) $2x^2 = 18$

$2x^2 = 18$

$x^2 = \frac{18}{2}$

$x = \sqrt{9}$

$x = 3$

c) $p^2 + 3 = 19$

$p^2 + 3 = 19$

$p^2 = 19 - 3$

$p^2 =16$

$p = \sqrt{16}$

p = 4

d) $m^2 + 5 = 9$

$m^2 + 5 = 9$

$m^2 = 9 - 5$

$m^2 = 4$

$$m = \sqrt{4}$$

$$\frac{2}{3}y^2 + 4 = 28$$

$$\frac{2y^2 + 12}{3} = \frac{84}{3}$$

$$2y^2 + 12 = 84$$

$$2y^2 = 84 - 12$$

e) $\frac{t^2}{2} = 50$

$$2y^2 = 72$$

$$\frac{t^2}{2} = 50$$

$$\frac{2y^2}{2} = \frac{72}{2}$$

$$t^2 = 50 \times 2$$

$$y^2 = 36$$

$$t^2 = 100$$

$$y = \sqrt{36}$$

$$t = \sqrt{100}$$

$$y = 6$$

$$t = 10$$

f) $\frac{2}{3}y^2 + 4 = 28$

Exercises.

Solve:

1) $4x^2 = 36$

5) $p^2 - 7 = 18$

9) $n^2 - 4 = 42$

2) $3p^2 = 243$

6) $6(x^2 + 2) = 306$

10) $t^2 = 567$

3) $\frac{1}{3}y^2 = 12$

7) $4(y^2 - 1) = 21$

11) $p^2 - 6 = 1290$

4) $b^2 + 3 = 12$

8) $2y^2 + 4 = 396$

12) $y^2 + 4 = 13$

6. Word problems involving equations.

1) Twice a number gives 16. Find the number.

Let x = the number

$$2x = 16$$

$$\frac{2x}{2} = \frac{16}{2}$$

$$x = 8$$

The number is 8

2) Four times a number added 6 gives 18. What is the number?

Let x = the number

$$4x + 6 = 18$$

$4x = 18 - 6$

$4x = 12$

$\dfrac{4x}{4} = \dfrac{12}{4}$

$x = 3$

The number is 3

3) Think of a number, multiply it by 5 then subtract 4 from the result, the answer is 26. What is the number?

Let y = the number

$5y - 4 = 26$

$5y = 26 + 4$

$5y = 30$

$\dfrac{5x}{5} = \dfrac{30}{5}$

$y = 6$

The number is 6

4) A third of a number subtracted from 10 gives 6. Find the number.

Let p = the number

$10 - \dfrac{p}{3} = 6$

$\dfrac{30-p}{3} = \dfrac{18}{3}$

$30 - p = 18$

$-p = 18 - 30$

$-p = -12$

$P = 12$

Exercises.

1) Twice a number plus 5 gives 17. Find the number.

2) The square of a number is equal to five times the number. What is the number?

3) When 4 is added to twice the square of a number, the result is 102. Find the number.

4) I think of a number, multiply it by 7 and add 3, I get 24. What number do I think of?

5) Four times a number is the same as the number increased by six. Find the number.

6) When a number is divided by 3 and add 4, the result is 7. Find the number.

7) When 2 is added to twice the square of a number, the result is 100. What is the number?

8) Think of a number, divide it by 2 and add 13, the result is 17. What is the number?

9) Five times a number added 2 the number is the same as six times a number subtracted 1. Find the number.

10) Thrice a number subtracted 4 gives 11. Find the number.

11) Two thirds of a number added 4 gives 10. What is the number?

12) A third of a number subtracted from 8 gives 3. Find the number.

13) Phocus is twice as old as Vincent. If their total age is 18 years. How old is each?

14) The sum of two numbers is 40 and their difference 20. What are the numbers?

15) When I subtract 1 from thrice a number, I get 20. Find the number.

16) The average of two numbers is 14. Find the two numbers if the small number is 4 less than the longer number.

7. More about problems involving equations.

1) The length of a rectangle is twice the width. If the perimeter is 24 cm, calculate the area.

$$\text{Let } x = W$$
$$2x = L$$
$$P = (L+W) \times 2$$
$$24 = (x + 2x) \times 2$$
$$24 = 2x + 4x$$

$$-2x - 4x = -24$$
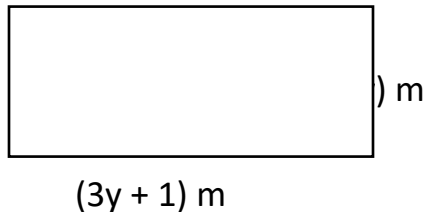$$-6x = -24$$
$$6x = 24$$
$$\frac{6x}{6} = \frac{24}{6}$$
$$x = 4$$

W=x = 4 cm

L=2x = 2 × 4cm = 8cm

A= L × W = 8cm × 4cm = **32 cm²**

2) If the perimeter of the figure below is 32 m, find the area.

) m

(3y + 1) m

P=(L+W) × 2

32 =(3y + 1+ 2y) × 2

32 = 6y + 2 + 4y

$$-6y - 4y = -32 + 2$$

$$-10y = -30$$

$$10y = 30$$

$$\frac{10y}{10} = \frac{30}{10}$$

y = 3

W = 2y = 2×3m = 6m

L = 3y + 1 = (3 × 3) + 1 = 9 + 1 = 10 m

A = L × W = 10m × 6m = **60m²**

Exercise.

1) The length of a rectangle is thrice the width. If the area is 48 m², find the perimeter.

2) The length of a rectangle is 4 times the width. If the perimeter is 400 cm, find the area.
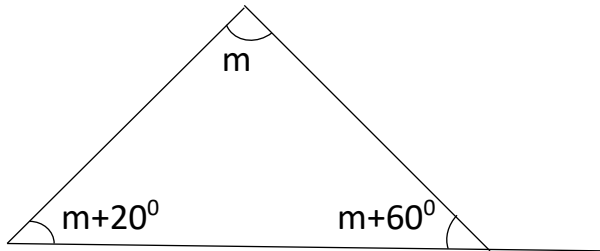
3) The width of a rectangle is 3m shorter than the length. If the perimeter is 26m, find its area.

4) The length of a rectangle if 4 cm longer than the width and the area is 140 cm^2. Find the perimeter.
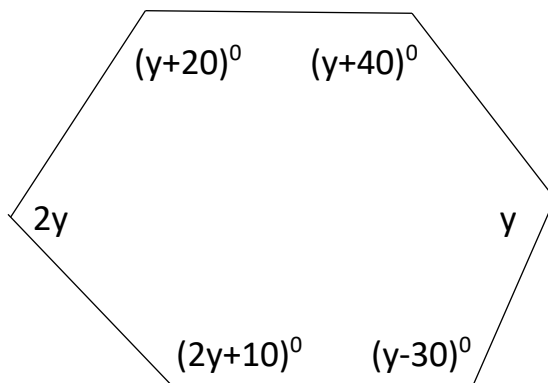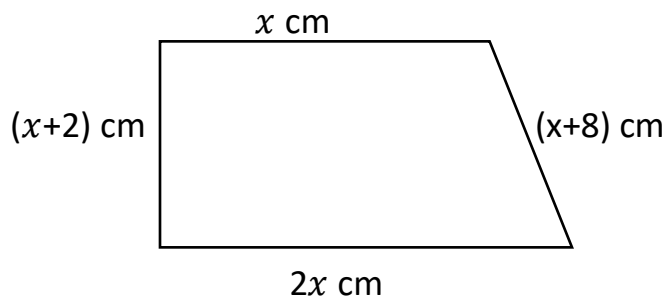
5) Find the value of x

x

$2x+10^0$ $x+20^0$
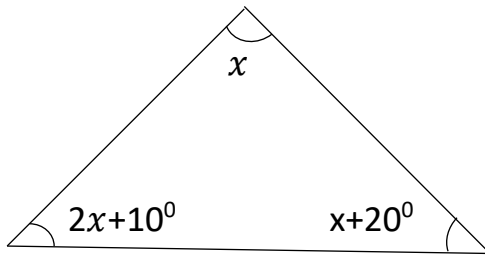
6) Calculate the size of angle m.

m

$m+20^0$ $m+60^0$

7) Calculate the size of each angle.

$(y+20)^0$ $(y+40)^0$

$2y$ y

$(2y+10)^0$ $(y-30)^0$

8) If the perimeter of the figure below is 30m, find its area.

x cm

$(x+2)$ cm $(x+8)$ cm

$2x$ cm

9) If the perimeter of the following figure is 35m find its area.

(p+10)m

(p+4)m

(p+6)m

10) Find the value of x

$5x$-5cm $2x$+4cm
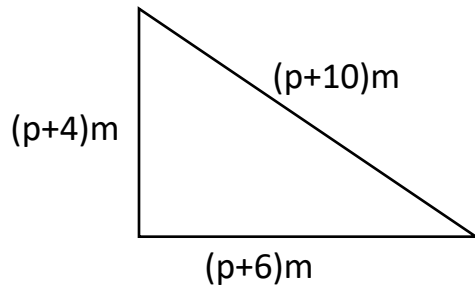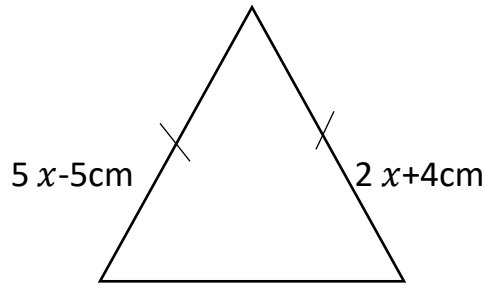
11) Find the area and perimeter of the figure below.

$(2x$-5) m

x m

$(x$+3) m
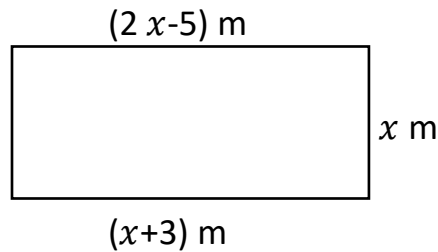
12) If the perimeter of this figure is 30 m, find the area.

$(2x$+4) cm

$\frac{x+3}{2}$ cm
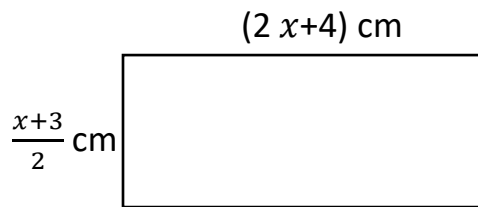
13) Calculate the value of y.

$(y+10)^0$ $(3y+20)^0$
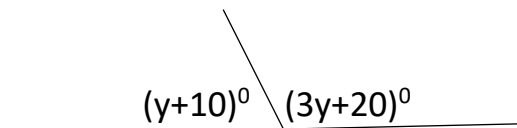
14) Study the diagram below.

   a) Find the value of x

   b) Find the value of y

$(x+80)^0$

y

$(2x+20)^0$

15) Find the value of x

$3x-10^0$

$2x+30^0$

16) Find the value of y

$3y-30^0$

$2y+30^0$ $y+80^0$
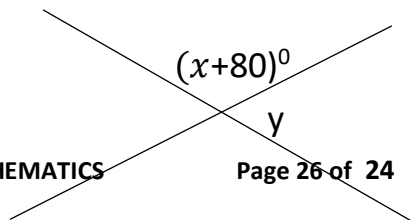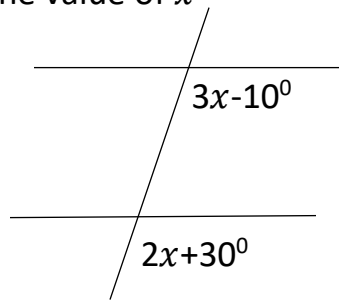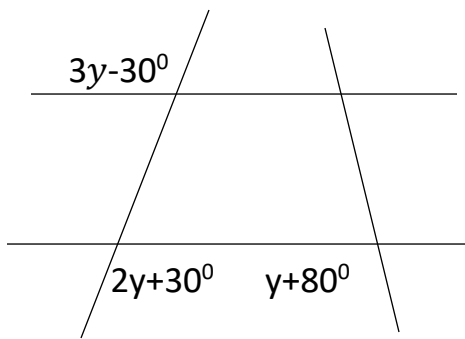
17) Find the size of each angle.

p

2p $p+20^0$
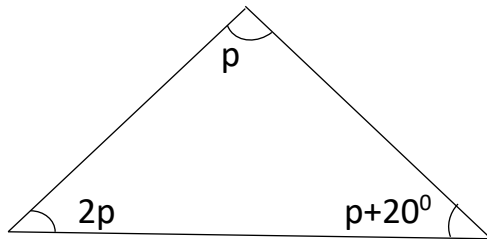
18) Study the figure below and find the value of x.

$x+10^0$

$x+20^0$

19) Observe the following figure.

$(4x-2)$ cm

$(2x+4)$ cm

h

PRIMARY SIX MATHEM/   FL Eduportal
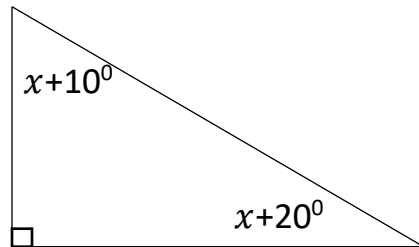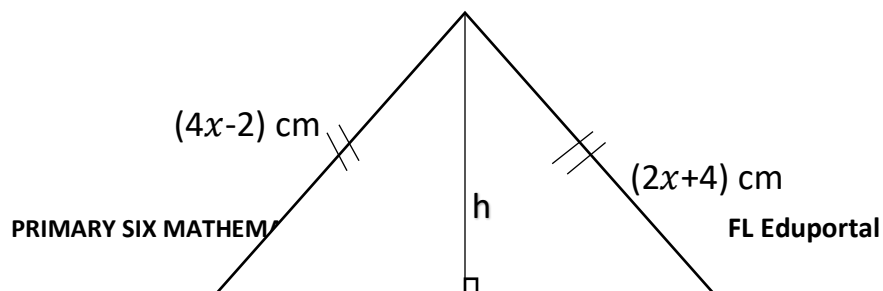
$(5x+1)$ cm

a) Find the value of x
b) Find the perimeter.
c) Calculate the value of h.
d) Calculate the area.

20) If the perimeter of the figure below is 46 m, find its area.
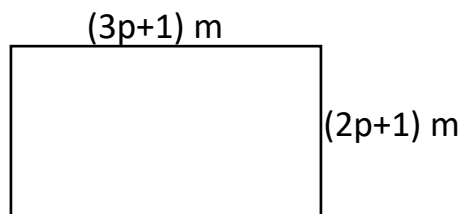
$(3p+1)$ m

$(2p+1)$ m

21) Find the area and perimeter of the figure below.

$(3p+4)$ cm

$(2p+2)$ cm

$(5p-2)$ cm

22) Find the size of each angle.

$3x+11^0$

$2x+11^0$ $2x+34^0$

$3x+4^0$ $2x$
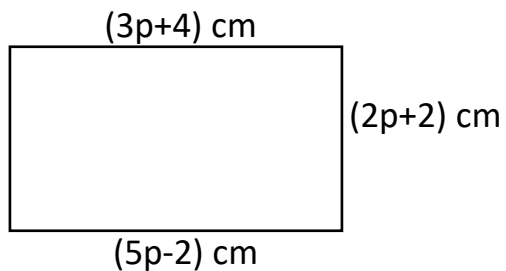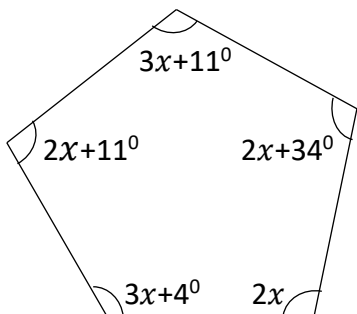
8. Consecutive numbers.

a) Consecutive counting numbers.

1) The sum of three consecutive counting numbers is 36. Find the numbers.

Let x = 1ˢᵗ number

$x+1$ = **2ⁿᵈ number**

$x+2$ = **3ʳᵈ number**

$3x + 3 = 36$

$3x = 36 - 3$

$3x = 33$

$$\frac{3x}{3} = \frac{33}{3}$$

$x = 11$

1ˢᵗ number = x = 11

2ⁿᵈ number = x +1 = 11 + 1=12

3ʳᵈ number = $x+2$ = 11+ 2 = 13

2) The sum of four consecutive numbers is 46. What are the numbers?

Let x = 1ˢᵗ number

$x+1$ = **2ⁿᵈ number**

$x+2$ =**3ʳᵈ number**

$x+3$ = **3ʳᵈ number**

$4x + 6 = 46$

$4x = 46 - 6$

$4x = 40$

$$\frac{4x}{4} = \frac{40}{4}$$

$x = 10$

1ˢᵗ number = x = 10

2ⁿᵈ number = x +1 = 10 + 1=11

3ʳᵈ number = $x+2$ = 10+ 2 = 12

4ᵗʰ number = $x+3$ = 10+ 3 = 13

Exercises.

1) The sum of three consecutive numbers is 72. Find the three numbers.

2) The sum of three consecutive numbers is 21. What are the numbers?

3) The sum of four consecutive numbers is 166. Find the numbers.

4) If the sum of three consecutive numbers is 42, find the numbers.

b) Consecutive even numbers.

1) The sum of three consecutive even numbers is 24. Find the numbers.

Let x = 1ˢᵗ number

x+2 = 2ⁿᵈ number

x+4 = 3ʳᵈ number

3x + 6 = 24

3 x = 24 – 6

3 x = 18

$$\frac{3x}{3} = \frac{18}{3}$$

x = 6

1ˢᵗ number = x = 6

2ⁿᵈ number = x + 2 = 6 + 2 = 8

3ʳᵈ number = x+4 = 6 + 4 = 10

2) The sum of three consecutive even numbers is 60.

Let x = 1ˢᵗ number

x+2 = 2ⁿᵈ number

x+4 = 3ʳᵈ number

3x + 6 = 60

3 x = 60 – 6

3 x = 54

$$\frac{3x}{3} = \frac{54}{3}$$

x = 18

1ˢᵗ number = x = 18

2ⁿᵈ number = x + 2 = 18 + 2 = 20

3ʳᵈ number = x+4 = 18 + 4 = 22

Exercises

1) The sum of three consecutive even numbers is 30. Find the numbers.

2) Thomas added three consecutive numbers and got 78. What were the numbers?

3) The sum of three consecutive even numbers is 36. Find the numbers.

4) Find the four consecutive even numbers whose sum is 172.

5) Given that the sum of three consecutive even numbers is 192, calculate the three numbers.

c) Consecutive odd numbers.

1) The sum of three consecutive odd numbers is 45. Find the numbers.

Let x = 1ˢᵗ number

x+2 = 2ⁿᵈ **number**

x+4 = 3ʳᵈ **number**

3x + 6 = 45

3 x = 45 – 6

3 x = 39

$$\frac{3x}{3} = \frac{39}{3}$$

x = 13

1ˢᵗ number = x = 13

2ⁿᵈ number = x + 2 = 13 + 2 = 15

3ʳᵈ number = x+4 = 13 + 4 = 17

2) David added three consecutive odd numbers and got 135. What were the numbers?

Let x = 1ˢᵗ number

x+2 = 2ⁿᵈ **number**

x+4 = 3ʳᵈ **number**

3x + 6 = 135

3 x = 135 – 6

3 x = 129

$$\frac{3x}{3} = \frac{129}{3}$$

x = 43

1ˢᵗ number = x = 43

2ⁿᵈ number = x + 2 = 43 + 2 = 45

3ʳᵈ number = x+4 = 43 + 4 = 47

Exercises.

1) The sum of three consecutive odd numbers is 57. What are the numbers?

2) The sum of three consecutive odd numbers is 42. Find the numbers.

3) Find three consecutive odd numbers whose sum is 51.

4) Yesterday Ana added three consecutive odd numbers and obtained 111. What were the numbers?

3) The sum of four consecutive odd numbers is 216. Find the numbers.

Average of consecutive numbers.

1) The average of three consecutive numbers is 14. Find the three numbers.

Let x = 1ˢᵗ number

x+1 = 2ⁿᵈ **number**

x+2 = 3ʳᵈ **number**

$\dfrac{3x+3}{3} = 14$

$3x + 3$ = 42

$3x$ = 42 − 3

$3x$ = 39

$\dfrac{3x}{3} = \dfrac{39}{3}$

x = 13

1ˢᵗ number = x = 13

2ⁿᵈ number = x + 1 = 13 + 1 = 14

3ʳᵈ number = x + 2 = 13 + 2 = 15

2) The average of three consecutive even numbers is 16. What are the numbers?

Let x = 1ˢᵗ number

x+2 = 2ⁿᵈ **number**

x+4 = 3ʳᵈ **number**

$\dfrac{3x+6}{3} = 16$

$3x + 6$ = 48

$3x$ = 48 − 6

$3x$ = 42

$\dfrac{3x}{3} = \dfrac{42}{3}$

x = 14

1ˢᵗ number = x = 14

2ⁿᵈ number = x + 2 + 14 + 2 = 16

3ʳᵈ number = x + 4 = 14 + 4 = 18

Exercises.

1) The sum of three consecutive odd numbers is 15. Find the numbers.
2) The average of four consecutive even numbers is 9. What are three numbers?
3) Moses found the average of three consecutive numbers and got 11. What were the numbers?

3) The average of three consecutive odd numbers is 21. Find the numbers.

TOPIC2: <u>SIMULTANEOUS EQUATIONS</u>

❖ Simultaneous equation is a system of equations where there are two or more unknowns in one equation. In simultaneous equations, two equations must appear.

Example: $\begin{cases} x + y = 11 \\ x - y = 1 \end{cases}$

The above two equations are both true at the same time. Then x has only one value and y has only one value as well.

<u>Examples</u>

Solve:

1) $\begin{cases} x + y = 11 \\ x - y = 1 \end{cases}$

$\begin{cases} x + y = 11 \\ x - y = 1 \end{cases}$

$2x = 12$

$\frac{2x}{2} = \frac{12}{2}$

$x = 6$

$\rightarrow x + y = 11$

$6 + y = 11$

$y = 11 - 6$

$y = 5$

2) $\begin{cases} 2x + y = 5 \\ x - 2y = 0 \end{cases}$

$\begin{cases} 2x + y = 5 & \times 2 \\ x - 2y = 0 & \times 1 \end{cases}$

$\begin{cases} 4x + 2y = 10 \\ x - 2y = 0 \end{cases}$

$5x = 10$

$\frac{5x}{5} = \frac{10}{5}$

$x = 2$

$\rightarrow x - 2y = 0$

$2 - 2y = 0$

$-2y = 0 - 2$

$-2y = -2$

$\frac{-2y}{-2} = \frac{-2}{-2}$

$y = 1$

3) $\begin{cases} 2x + 3y = 22 \\ 4x - 2y = 12 \end{cases}$

$\begin{cases} 2x + 3y = 22 & \times 2 \\ 4x - 2y = 12 & \times 3 \end{cases}$

$\begin{cases} 4x + 6y = 44 \\ 12x - 6y = 36 \end{cases}$

$16x = 80$

$\frac{16x}{16} = \frac{80}{16}$

$x = 5$

$\rightarrow 2x + 3y = 22$

$2 \times 5 + 3y = 22$

$10 + 3y = 22$

$3y = 22 - 10$

$3y = 12$

$\frac{3y}{3} = \frac{12}{3}$

$y = 4$

4) $\begin{cases} 3x + y = 11 \\ x + 2y = 7 \end{cases}$

$\begin{cases} 3x + y = 11 & \times -2 \\ x + 2y = 7 & \times 1 \end{cases}$

$\begin{cases} -6x - 2y = -22 \\ x + 2y = 7 \end{cases}$

$-5x = -15$

$\frac{-5x}{-5} = \frac{-15}{-5}$

$x = 3$

$\rightarrow x + 2y = 7$

$3 + 2y = 7$

$2y = 7 - 3$

$2y = 4$

$y = 2$

<u>Exercises.</u>

Solve:

1) $\begin{cases} x + 2y = 5 \\ 2x + y = 4 \end{cases}$

2) $\begin{cases} 3x + y = 11 \\ x + 2y = 7 \end{cases}$

3) $\begin{cases} 2x + 3y = 23 \\ 3x + 2y = 22 \end{cases}$

4) $\begin{cases} 5x + 2y = 26 \\ 3x - 3y = 3 \end{cases}$

5) $\begin{cases} x + 2y = 6 \\ 3x - y = 4 \end{cases}$

6) $\begin{cases} 2x + 3y = 8 \\ 3x + 4y = 11 \end{cases}$

7) $\begin{cases} 3x - 2y = 24 \\ x + y = 8 \end{cases}$

World problems involving simultaneous equations.

1) Alex bought 3 pens and 2 books at 280 F altogether. The next day he bought 1 pen and 2 books at 160 F altogether. Calculate the price of 1 pen and 1 book.

Let x=the cost of 1 book

y=the cost of 1 pen

$\begin{cases} 2x + 3y = 280 \quad \times 1 \\ 2x + y = 160 \quad\quad \times -3 \end{cases}$

$\begin{cases} 2x + 3y = 280 \\ -6x - 3y = -480 \end{cases}$

$-4x = -200$

$\dfrac{-4x}{-4} = \dfrac{-200}{-4}$

$x = 50$ **Frw**

$\rightarrow 2x + y = 160$

$2 \times 50 + y = 160$

100 + y = 160

y = 160 − 100

y = 60 Frw

The cost of 1 pen = 50 Frw

The cost of 1 book = 60 Frw

2) A sum of 56,000 F is composed of 2,000 F notes and 500 F notes only. If there are 37 notes altogether, find the number of notes of each type.

Let x= 2,000 F notes

y= 500 F notes

$\begin{cases} x + y = 37 \\ 2000x + 500y = 56000 \end{cases}$

$\begin{cases} x + y = 37 \quad \times -1 \\ 4x + y = 112 \quad\quad \times 1 \end{cases}$

$\begin{cases} -x - y = -37 \\ 4x + y = 112 \end{cases}$

$3x = 75$

$\dfrac{3x}{3} = \dfrac{75}{3}$

$$x = 25$$
$$\rightarrow x + y = 37$$
$$25 + y = 37$$
$$y = 37 - 25$$
$$y = 12$$

Exercises

1) 20 men and 30 children eat 110 kg of rice in a week while 40 men and 50 children eat 240 kg of rice in a week. How many kg does 1 man and 1 child eat?

2) Eight hens and three rabbits cost 79,000 Frw altogether while five rabbits and four hens cost 57,000 Frw. Find the cost of each animal.

3) A sum of 1,180F is made up of 50F coins and 20F coins. If there are 32 coins altogether, find the number of coin of each type.

4) Peter bought 5 cows and 3 bulls at 1,900,000F. He bought again 3 cows and 2 bulls at 1,200,000F. Find the cost of 1 cow and 1 bul.

5) Anita received her salary composed of 50F coins and 100F coins. If she received 13,500F and 160 coins, how many 50F coins and 100F coins did she receive?

6) 60 boys and 80 girls dig 320m while 20 boys and 40 girls dig 140m. How many metres does 1 boy dig and how many metres does 1 girl dig?

7) At Amahoro stadium there are 24,000 seats. After a match, they collected 17,000,000F for males and females. If a male fun pays 1,000F and a female one pays 500f, how many boys and girls watched the match?

8) Kalisa bought 50 pens and 60 pencils at 8,000F. The following day he bought 80 pens and 40 pencils at 10,000F. What is the cost of 1 pen and 1 pencil?

9) 3 kg of beans and 2 kg of salt cost 1,700F while a kilo of salt and five kg of beans cost 2,250F. Find the unit price of 1 kg of each.

10) Mutesi bought 2 kilograms of sugar and 3 kilograms of rice at 3,450F. The following day she bought 3 kilograms of sugar and 4 kilograms of rice at 4,88F. Find the unit price of 1 kg of each type.

11) A sum of 16,000 Frw is composed of 500F notes and 1000F notes. If there are 22 notes altogether, find the number of notes of each type.

12) A sum of 90,000F is made up of 2,000F notes and 5,000F notes. If the total number of notes is 30, find the number of notes of each type.

13) 3 books and 2 pens cost 230F while 2 books and 1 pen cost 140F. what is the cost of each?

TOPIC 3: <u>INEQUALITIES</u>

Relationships between two identical objects with differences in one way or another can be described by use an inequality
The number can be compared easily by describing their locations on the number line.

a) <u>Introduction</u>

Consider the following expressions.

a) $1 < 7$

d) $\frac{1}{3} > \frac{1}{2}$

b) $-2 > -5$
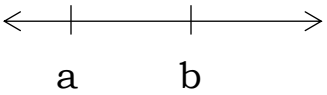
e) $x - 1 \leq 5$
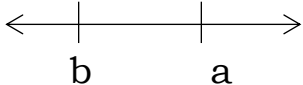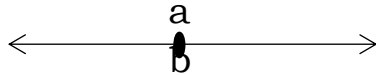
c) $8 \leq -14$

f) $2y + 3 > -4$

- All these expressions are **inequalities.**
- Inequalities have two sides: –The left hand side

           –The right hand side

- The two sides of inequalities are separated by an **inequality sign.**

<u>Phrase</u>	<u>Symbol</u>
Is less than	$<$
Is greater than	$>$
Is less than or equal to	$\leq$
Is greater than or equal to	$\geq$
Is not equal to	$\neq$
Is equivalent to	$\Longleftrightarrow$

Symbol	Words	Number line
$a < b$	a is less than b	a lies to the left of b
		←——+———+——→
		a b

a> *b*	a is greater than b	a lies on the right of b ←———|———|———→ b a

a= *b*	a is equal to b	a and b are at the same point
		      a
		 ←———●———→
		      b

When number lies between two other numbers ,we may use a three –part inequality to express that fact.

e.g : To say that 5 is between 2 and 9 we write; $2< 5 < 9$

This inequality says that 5 is greater than 2 and less than 9.

EXERCISE

1. Write an inequality to say that 0 is between -1 and 3

2. Write an inequality to say that 4 is between 3 and 10

3. Write an inequality to say that -2 is between -3 and 2

4. Write an inequality to say that -5 is between 0 and 1

a. WRITING SOLUTIONS OF INEQUALITY

Solution: is a set of all possible values to the unknown letter.

→The solution of x may be required from

- **Integers48[pip'up[iuoi-[-]\0p'**
- **];o666**
- **Whole numbers**
- **Counting numbers**

- **Given range.etc**

 Examples

- In expressions **(a)** and **(b)**, the inequalities are verified (**are true**).
- In expressions **(c)** and **(d)**, the inequalities are not verified (**are false**).
- In expressions **(e)** and **(f)**, the inequalities are neither true nor false. The truth or untruth depends on the value given to x and **y**.

Note: Expressions **(e)** and **(f)** are called **inequalities.**

b) Explaining inequalities

1) The distance between two villages is more than 18 km. Write this as an algebraic statement.

$$\rightarrow x > 18km \text{ when } x \text{ represents the distance}$$

2) I have x **Frw.** I spend 5,000Frw. The amount I have left with is less than 2,000Frw. Write an inequality using x**.**

$$\rightarrow x - 5,000Frw < 2,000Frw$$

3) I had x Frw**.** Peter gave me 500Frw. The amount of money I have is greater 1,500Frw. Express it as an inequality.

$$\rightarrow x + 500Frw > 1,500Frw$$

c. Representing inequalities on a number line

1) Represent $x < 3$ on a number line.

-7 -6 -5 -4 -3 -2 -1 0 +1 +2 +3 +4 +5 +6 +7
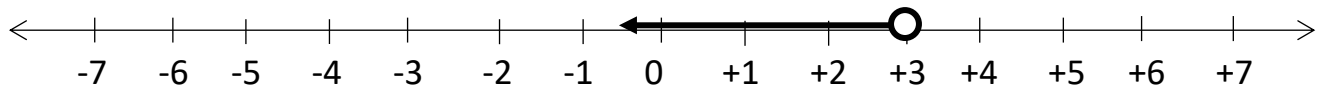
This means x can have any value less than 3

- ❖ The arrow shows the range of the values that x can have.
- ❖ The empty circle at 3 shows that the value 3 is not included as the one of the value of x.

2) Represent $x > -2$ on a number line.

-7 -6 -5 -4 -3 -2 -1 0 +1 +2 +3 +4 +5 +6 +7
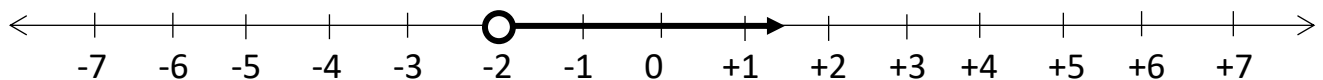
3) Represent $x \leq 4$ on a number line.
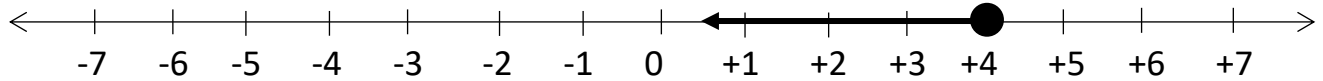
+7**Notice:** the shaded circle at 4 shows that the value 4 is one of the values of x.

4) Represent $x \geq -3$ on a number line.
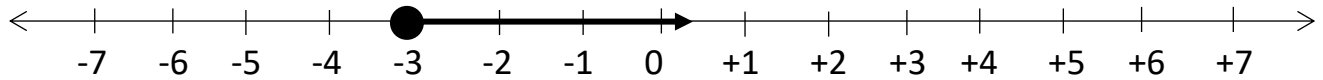
d) Discovering the inequalities shown on a number line.

Write the inequalities shown on the following number lines.

1)
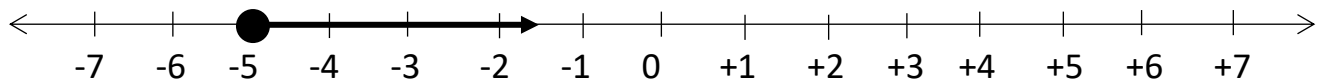
$x \geq -5$

2)
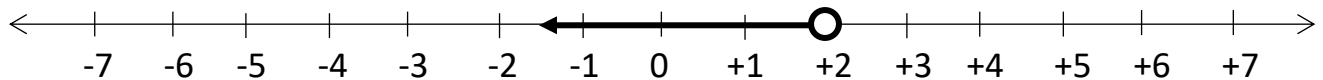
$x < +2$

3)
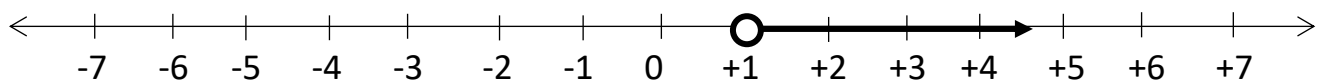
$x > +1$

d) Solving inequalities

Solving an inequality involves finding all the value of the unknown that can make the inequality true.

The values are called **solution of the inequality.**

The solution set is, in general, expressed as an interval.

Exercises

Solve the following:

1) $5 - 4x \geq -x + 8$

$$5 - 4x \geq -x + 8$$

$$-4x + x \geq +8 - 5$$

$$-3x \geq +3$$

$$\frac{-3x}{-3} \geq \frac{+3}{-1}$$

$$x \leq -1$$
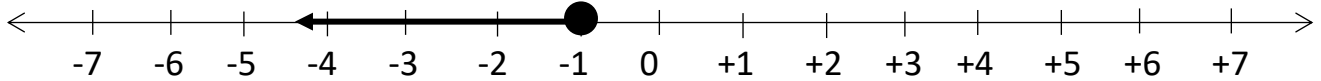
S={-1, -2, -3, …….}

2) $\frac{1}{3}x - (x + 1) \geq 3$

$$\frac{1}{3}x - (x + 1) \geq 3$$

$$\frac{x - 3(x+1)}{3} \geq \frac{9}{3}$$

$$x - 3(x + 1) \geq 9$$

$$x - 3x - 3 \geq 9$$

$$x - 3x \geq 9 + 3$$

$$-2x \geq 12$$

$$\frac{-2x}{-2} \geq \frac{12}{-2}$$

$$x \leq -6$$
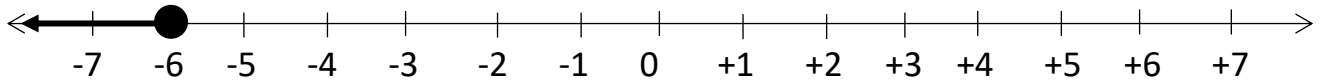
S={-6, -7, …….}

3) 2y + 5 > 11

2y + 5 > 11

2y > 11 − 5

2y > 6

$$\frac{2y}{2} > \frac{6}{2}$$

y > 3
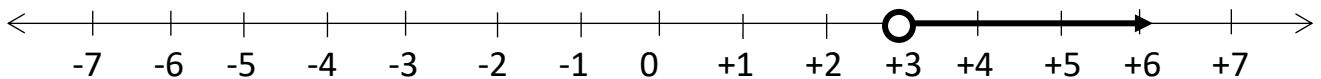
S={+3, +4, +5, …….}

4) 3p − 1 > p + 7

$$3p - 1 > p + 7$$
$$3p - p > 7 + 1$$
$$2p > 8$$
$$\frac{2p}{2} > \frac{8}{2}$$
$$p > 4$$

$$S=\{+4, +5, +6, \ldots\ldots\}$$
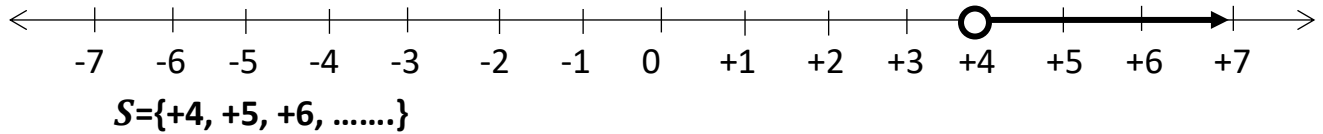
EXERCISES

Solve:

1) $5x + 5 < x + 6$

2) $2x + 4 \geq 4x + 10$

3) $\frac{x}{2} - 1 < 3$

4) $x + 2 > 3x$

5) $4y - 3 \leq 2y - 7$

6) $2m + 3 > 5$

7) $3(x + 5) < 2(x + 8)$

8) $2(x - 3) + 4 \geq -(2x + 2)$

9) $\frac{4n+7}{3} \leq \frac{6n+8}{4}$

10) $\frac{x+1}{2} > \frac{x-1}{3}$

11) $3 + \frac{2y}{3} < 5$

12) $4p + 1 \leq 9$

13) $3x + 4 > 6x - 5$

14) $5y + 2 \geq 2y + 6$

e) Word problems involving inequalities.

1) If 3 is added to the double of a number, the result is less than 7 added to the number. Express it as an inequality and solve.

Let x = the number

$$2x + 3 < x + 7$$
$$2x - x < 7 - 3$$
$$x < 4$$

$$S=\{+4, +3, +2, \ldots\ldots\}$$
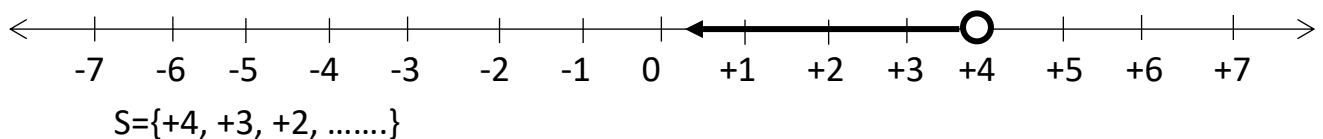
2) If 2 is added to the number, the result is greater than or equal the triple of a number. Express it as an inequality and solve.

n**Let** x = **the number**

$$x + 2 \geq 3x$$
$$x - 3x \geq -2$$
$$-2x \geq -2$$
$$\frac{-2x}{-2} \geq \frac{-2}{-2}$$
$$x \leq 1$$

S={+1, 0, -1, ...}
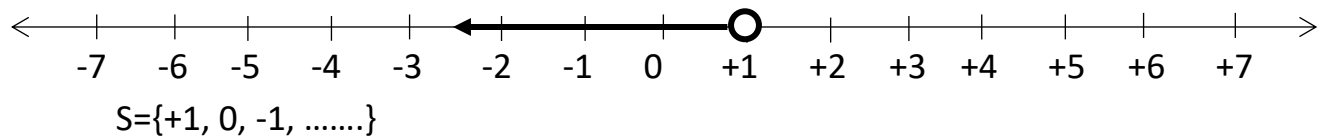
Exercises

1) I think of a number, divide it by 4 and the answer is greater or equal to 2. Express it as an inequality and solve.

2) Four times a number added 5 is less than the number added 14. Express it as an inequality and solve.

3) Thrice a number added to 2 is greater than 4 subtracted from six times the number. Express it as an inequality and solve.

4) Four times a number subtracted from 5 is less than or equal to 5 subtracted from the number. Express it as an inequality and solve.

TOPIC 4: <u>WHOLE NUMBERS</u>

I. <u>NUMERATION</u>

1) <u>Place value of digits</u>

1) What is the place value pf each digit in the number 147,306,429,185?

Solution:

Billions			Millions			Thousands			Units		
H	T	O	H	T	O	H	T	O	H	T	O

2	4	7	3	0	6	4	2	9	1	8	5

Ones

Tens

Hundreds

Thousands

Ten thousands

Hundred thousands

Millions

Ten millions

Hundred millions

Billions

Ten billions
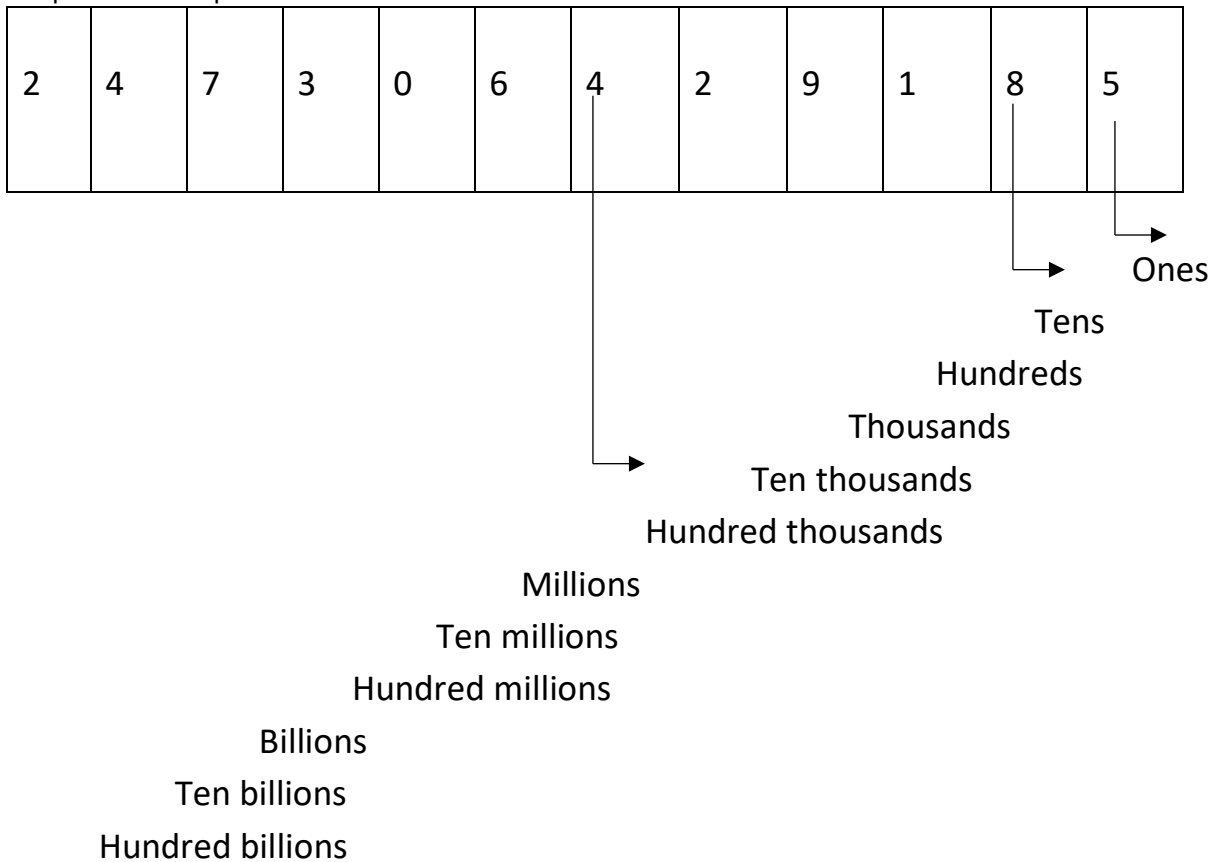
Hundred billions

2) What is the place value of 3 in the number 243,109,764?

243,109,764

└──►**Millions**

3) What is the rank of 4 in the number 2,468,903,127?

2,468,903,127

└──►**Hundred millions**

4) Write the position of 6 in the number 46,219.

46,219

└──►**Thousands**

Note: | Place value = Rank = Position

3) **Value of digits**.

To get the value of any digit, multiply it by its place value.

Examples.

1) Find the value of each digit in the number 273,109,864,529.

Solution:

Number	Place value	Value
273,109,864,529	Ones	$7 \times 1 = 7$
	Tens	$2 \times 10 = 20$
	Hundreds	$5 \times 100 = 500$
	Thousands	$4 \times 1,000 = 4,000$
	Ten thousands	$6 \times 10,000 = 60,000$
	Hundred thousands	$8 \times 100,000 = 800,000$
	Millions	$9 \times 1,000,000 = 1,000,000$
	Ten millions	$0 \times 10,000,000 = 0$
	Hundred millions	$1 \times 100,000,000 = 200,000,000$
	Billions	$3 \times 1,000,000,000 = 3,000,000,000$
	Ten billions	$7 \times 10,000,000,000 = 30,000,000,000$
	Hundred billions	$2 \times 100,000,000,000 = 200,000,000,000$

2) What is the value of 4 in 346,290,127?

346,290,127

$4 \times 10,000,000 = 40,000,000$

3) Find the sum of the values of 3 and 5 in the number 23,659,186.

23,659,186

Value of 5 = 5 $\times$ 10,000 = 50,000

Value of 3 = 3 $\times$ 1,000,000 = 3,000,000

Sum = 3,000,000 + 50,000 = 3,050,000

4) Calculate the difference between the value of 2 and 8 in the number 48,917,253.

Value of 2 = 2 $\times$ 100 = 200

Value of 8 = 8 $\times$ 1,000,000 = 8,000,000

Difference = 8,000,000 – 200 = 7,999,800

5) Find the quotient of values of 6 and 3 in the number 68,236,197.

Value of 6= 6 $\times$ 10,000,000 = 60,000,000

Value of 8 = 3 $\times$ 10,000 = 30,000

Sum = 60,000,000 $\div$ 30,000 = 2,000

3) <u>Reading and writing numbers in words</u>.

Observe the following numbers written in a table and write them in words.

Billions			Millions			Thousands			Units		
		2	0	0	4	6	7	0	2	0	0
			7	0	0	0	7	0	0	0	7
2	5	0	1	0	9	6	3	7	3	9	0
					3	8	0	2	7	1	6

Solution:
- **2,004,670,200 = two billion, four million, six hundred seventy thousand two hundred.**
- **700,070,007 = seven hundred million, seventy thousand, seven.**
- **250,109,637,390 = two hundred fifty billion, one hundred nine million, six hundred thirty seven thousand, three hundred ninety.**
- **3,802,716 = three million, eight hundred two thousand seven hundred sixteen.**

4) <u>Writing numbers in figures</u>.
Write the following numbers in figures:
1) Sixty billion, six hundred million, six thousand, sixty.

Billions			Millions			Thousands			Units		
H	T	O	H	T	O	H	T	O	H	T	O

6	0	6	0	0	0	0	6	0	6	0

60,600,006,060

2) Seven million, seventy thousand, seven hundred.

Billions			Millions			Thousands			Units		
H	**T**	**O**	**H**	**T**	**O**	**H**	**T**	**O**	**H**	**T**	**O**
				7	0	0	7	0	7	0	0

7,070,700

Exercises

1) What is the position of 7 in the number 37,089,231?

2) Give the place values of the underlined digits.

    a) 646, 290,317

    b) 231,038,478

    c) 56,251,029

3) Find the value of 6 in the number 346,098,124.

4) What are the place value and the value of 7 in the number 276,109,841?

5) What are the place value of 2 and the value of 9 in the number 269,208,465?

6) Write the following numbers in words:

    a) 2,009

    b) 8,000,0008

    c) 11,101,111

    d) 84,678,239

7) Write the numbers below in figures.
- a) One thousand, nine hundred fifty five
- b) Four hundred thirty million, six hundred forty.
- c) Twenty million, four thousand, nine hundred.
- d) Eighty billion and two million and

8) Find the sum of the values of 5 and 7 in the number 254,720,189.

9) Calculate the difference between the values of 6 and 4 in the number 24,068,915.

10) What is the product of the values of 3 and 9 in the number 23,597?

11) Find the quotient of the values of 9 and 3 in the number 436,859,102,847.

5) EXPANDED FORM

a) Expanding in place value form.

Expand the numbers in place value form.

1) 245,304 =(2 × 100,000) + (4 × 10,000) + (5 × 1,000) + (3 × 100) + (0 × 10) + (4 × 1)

2) 350,841,256 = (3×100,000,000) + (5×10,000,000) + (0×1,000,000) + (8×100,000) +
      (4×10,000) + (1×1,000) + (2×100) + (5×10) + (6×1)

b) Expanding in multiples of 10.

Expand the numbers below in multiples of 10.

1) 324,689 = (3 ×100,000) + (2×10,000) + (4×1,000) + (6×100) + (8×10) + (9×1)
    = (3×10×10×10×10×10) + (2×10×10×10×10) + (4×10×10×10) +
    (6× 10 ×10) + (8×10) + (9×10)

2) 48,901 = (4×10,000) + (8×1,000) + (9×100) + (0×10) + (1×1)
    = (4×10×10×10×10) + (8×10×10×10) + (9× 10 ×10) + (0×10) + (1×10)

c) Expanding in value form.

Expand these numbers in value form.

1) 2,678,045 = (2×1,000,000) + (6×100,000) + (7×10,000) + (8×1,000) + (0×100) +
    (4×10) + (5×1)

= 2,0000,000 + 600,000 + 70,000 + 8,000 + 0 + 40 + 5

3) 456,109,867 = (4×100,000,000) + (5×10,000,000) + (6×1,000,000) + (1×100,000) +

(0×10,000) + (9×1,000) + (8×100) + (6×10) + (7×1)

= 400,000,000 + 50,000,000 + 6,000,000 + 100,000 + 0 +9,000 + 800 +

60 + 7

d) Expanding in power form.

Expand the following numbers in power form.

1) 2,430,798 = (2×1,000,000) + (4×100,000) + (3×10,000) + (0×1,000) + (7×100) +

(9×10) + (8×1)

$= (2 \times 10^6) + (4 \times 10^5) + (3 \times 10^4) + (0 \times 10^3) + (2 \times 10^2) + (9 \times 10^1) + (8 \times 10^0)$

2) 425,769 = (4×100,000) + (2×10,000) + (5×1,000) + (7×100) +

(6×10) + (9×1)

$= (4 \times 10^5) + (2 \times 10^4) + (5 \times 10^3) + (7 \times 10^2) + (6 \times 10^1) + (9 \times 10^0)$

5) Finding the expanded number

1) Which number has been expanded to give the following?

$(4 \times 10^5) + (2 \times 10^4) + (5 \times 10^3) + (7 \times 10^2) + (6 \times 10^1) + (9 \times 10^0)$

      400,000
       20,000
  +   5,000
          700
           60
  _____
            9
      425,769

2) Find the number that was expanded to become 500,000 + 40,000 + 2,000 + 600 + 0 +

8

      500,000
       40,000
  +   2,000
          600
            0
  _____
            8
      542,608

EXERCISES

1) Expand the numbers below in value form.

   a) 43,908

   b) 234,618

   c) 4,089,356

2) Expand the following numbers in power form.

   a) 340,728

   b) 97,129,935

   c) 240,143,176

3) Peter added 543,829 and 245,108. Expand the answer he got I power form.

4) The population of Rwanda is 13,278,914. Expand the number is value form.

5) Which number has been expanded to give $(2\times10^5) + (8\times10^4) + (5\times10^3) + (0\times10^2) + (9\times10^1) + (1\times10^0)$?

6) Find the number that has been expanded to give 3,000,000 + 200,000 + 20,000 + 5,000 + 800 + 40 + 3.

7) Felix expanded a number and got $(9\times10^6) + (5\times10^5) + (2\times10^4) + (1\times10^3) + (8\times10^2) + (5\times10^1) + (7\times10^0)$

6) **Forming numerals from digits**

In Hindu Arabic numerals system there are 10 digits such as **1, 2, 3, 4, 5, 6, 7, 8, 9** and **0**.
All numbers are formed using these above digits.

Examples

1) Write down all numbers that can be formed from the digits 2, 7 and 5

     275   725   527

     257   752   572

2) Write down all numbers that can be formed using the digits 6, 4, 0 and 8

    6408   4068   8046

    6480   4086   8064

    6048   4806   8604

    6084   4860   8640

    6804   4608   8406

    6840   4680   8460

3) What is the smallest number that can be formed from the digits 8, 6, 0, 9 and 2?

    Smallest number= 20,689

4) Give the largest number that can be formed from the digits 5, 1, 7, 0, 6 and 4

    Largest number = 765,410

5) Find the sum of the lowest number and the biggest number that can be formed from the digits 4, 8, 0, 3, 6 and 7.

<div align="center">

Smallest number= 30,478

Largest number = 87,430

Sum = 30,478 + 87,430 =117,908

</div>

Exercises

1) Form two six digit numbers from the digits: 1, 2, 3, 4, 5 and 6.
2) Give the lowest seven digits number formed from 4, 3, 0, 8, 5, 1, 9.
3) Find the difference between the biggest number and the smallest number that can be formed using the digits 6, 2, 9, 0, 4 and 7.
4) What is the sum of the lowest number and the largest number formed from the digits 8, 1, 5, 0, 7, 3 and 9.
5) Write two even numbers that can be formed from the digits 7, 4 and 3.
6) Form the largest and the smallest number using the digits 6, 8, 2, 7 and 5.
7) Give any three odd numbers that can be formed from the digits 8, 3, 9 and 2.
8) Write down all numbers that can be formed using 4, 8 and 2.

8) Complements of numbers

A complementary number is a number that can be added to another to make a rounded figure.

A rounded figure is number that is equal to **10; 100; 1,000; 100,000; 1,000,000 etc**

- The complementary number to 4 is **6** because **4 + 6 = 10**
- The complement of 27 is **73** because **27 + 73 = 100**
- The complement of 516 is **484** because **516 + 484 = 1000**

Examples

Find the complements of the following numbers;

   a) 68 = **100 – 68 = 32**
   b) 451 = **1000 – 451 = 549**
   c) 2,891 = **10,000 – 2,891 = 7,109**
   d) 72 = **100 – 72 = 28**
   e) 4 = **10 – 4 = 6**

Exercises

1) Find the complements of the following numbers;

a) 47

b) 382

c) 19

d) 7

e) 7,148

f) 28,495

2) What is the complementary number of 467?

3) Find the complement of 1,748.

4) What should be added to 34,617 to give 50,000?

5) Robert had 482 Frw and wanted to buy a dozen of pens at 600 Frw. How much money does he need?

6) Agape planned to plant 10,000 seedlings. He only has 7,218 seedlings. How many more seedling does he need?

7) Calculate the complementary number to 57.

9) **Rounding off whole numbers**

➢ If the figure on the right of the required place value is less than 5 (0,1,2,3,4), add **0** to the required place value.

➢ If the figure on the right of the required place value is 5 or greater than 5 (5,6,7,8,9) add **1** to the required place value.

➢ All digits behind the required place value become zeros.

Examples

1) Round off 268 to the nearest tens.

> **2<u>6</u>8**
> **+1**
> **270**

2) Round off 3,749 to the nearest hundred.

> **3,<u>7</u>49**
> **+0**
> **3,700**

3) Round off 43,524 to the nearest thousand.

43,5̲24

 +1

44,000

4) Correct 173,897 to the nearest ten thousands.

17̲3,897

 +1

170,000

Exercises

1) Round off 39,621 to the nearest thousands.
2) Manzi bought a radio at 25,875 Frw. Round the amound of money to the nearest ten thousand.
3) Find on number that when rounded to the nearest hundreds is 453 000.
4) A number rounded to the nearest hundreds is 6,700.
    a) Determine the lowest possible number.
    b) Determine the highest possible number.
5) The average number of pupils in primary schools is 3,489,989. Round this number to the nearest thousand.
6) Agatesi bought her car for 9,561,000 Frw. Round off the money she paid to the nearest million.
7) Add 24,896 and 73,586 and round the result to the nearest thousand.

10) Comparing whole numbers using $<, >$ or =

When comparing we use:

➤ $>$: **greater than**

➤ $<$: **less than**

➤ $=$: **equal to**

To compare two or more numbers, first, count the number of digits in each number.

1) If the given numbers have different number of digits, the one having more digits is greater.

Examples

Compare the following numbers.

a) $46,798 < 123,012$

b) $23,401 > 9,876$

c) $238,012,741 > 89,376,874$

2) If the given numbers have the same number of digits, compare digit by digit from the left, until the two digits in the same corresponding place value tell which number is greater.

Examples

Compare the numbers below.

d) 274,692 < 274,892

a) 372,817,092 > 372,817,091

b) 845,127 = 845,127

Exercises

1) Write **true** or **false**.

a) 127,398 is less than 98,753

b) 381,645 is greater than 381,641

c) A number with more digits is greater than another number with less digits.

2) Fill in the spaces with the correct symbol of comparison.

a) 610 487 [_____] 7 248 160

b) 713 482 917 [_____] 713 482 907

c) 541 845 [_____] 541 845

3) Mutesi has 468 379 Frw and John has 98 476 Frw. Who has less money?

4) Which number is greater, 595 06 or 607 899? Explain.

5) Camille harvested 5,562 tonnes of beans and Felix harvested 5,256 tonnes of beans. Who harvested more beans?

6) Gasabo district collected 45,853,925 Frw in taxes while Gakenke collected 9,756,895 Frw. Which district collected less money?

11) Ordering numbers

Numbers are arranged in two ways;

1) **Ascending /increasing order:** means to arrange numbers from the smallest to the largest.

2) **Descending/decreasing**: means to arrange numbers from the largest to the smallest.

Examples

1) Arrange the following numbers in ascending order.

468 274 ; 468 374 ; 466 274 ; 468 284

Solution:

466 274; 468 274; 468 284; 468 374

2) Re-arrange the numbers below I decreasing order.

25 649; 25 639; 25 648; 256 481; 25 688

Solution:

256 481; 25 688; 25 649; 25 639

Exercises

1) Underline the smallest number.

33 333; 3 333; 333 333

2) Arrange the following numbers in ascending order.

a) 457 782; 457 792; 459 682; 98 765

b) 1,673,421; 1,065,345; 1,671,241; 1,065,234

c) 750,236,172; 750,237,172; 750,236,072; 750,236,174

3) Bank notes are numbered in order from A2408700 to A2408719.

a) How many notes are there altogether?

b) If each note is 5,000 Frw, how much money is in the bundle?

4) Order from the highest to the lowest.

a) 45,238; 110,210; 45,338; 45,239

b) 12,042,994; 12,420,994; 12,994,609; 12,499,906

12. OPERATIONS IN WHOLE NUMBERS

1) Addition

Addition basically means putting together.

When adding two or more numbers, first arrange digits in the numbers according to their place values (ones under ones, tens under tens, …….). Always start adding from ones and carry where necessary to the larger place value.

Examples

1) Add 346,897 and 49,736

$$\begin{array}{r} 346{,}897 \\ +49{,}736 \\ \hline 396{,}736 \end{array}$$

Note:

$$\begin{array}{r} 36 \longrightarrow Adding \\ +12 \longrightarrow Adding \\ \hline 48 \longrightarrow Sum \end{array}$$

2) A factory made 1,358,916 nails on Monday and 963,078 nails on Tuesday. Find the total numbers of nails which were made by the factory in two days.

$$\begin{array}{r} 1,358,916 \\ +963,078 \\ \hline 2,321,994 \end{array}$$

Exercises

1) Peter bought a radio at 434 890 Frw, a suit at 34 878 Frw and a book at 7 396 Frw. How much money did he spend altogether?

2) Betty deposited money in Umurenge SACCO as follows: 720,654 F in January, 1,004,529 F I February and 3,894,728 F in March.
   a) Calculate the total deposit Ingabire made in the Umurenge SACCO.
   b) Why is it necessary to deposit money in the bank?

3) What is the sum of 38,298,784 and 25,734,928?

4) Increase 5,702,854 by 4,589,627.

5) 4,836 and 1,689 make what?

6) A poultry farmer sold 252 797 chickens in one year. The next year he sold 391 358 chickens. The third year he sold 198 524 chickens. How many chickens did he sell in three years?

7) There were 246 240 books in a library and 167 645 more books were donated to the same library. How many books are there altogether?

8) Complete:

a)
$$\begin{array}{r} .\,8\,.\,9 \\ +3\,6\,9\,. \\ \hline 6\,.\,4\,2 \end{array}$$

b)
$$\begin{array}{r} 4\,6\,7\,. \\ +\,2\,.\,4\,6 \\ \hline .\,5\,.\,1 \end{array}$$

c)
$$\begin{array}{r} 1\,.\,3\,4 \\ +\,.\,7\,.\,6 \\ \hline 7\,6\,3\,. \end{array}$$

d) 68 + . = 152

e) . + 728 = 1,420

Properties of addition

1) Commutative property

The result after adding numbers in any order remains the same.

Examples

Add **a)** 20 + 40 =

Skill 1: 20 + 40 = 60

Skill 2: 40 + 20 = 60

Therefore 20 + 40 = 40 + 20

b) 70 + 30 = 100

Skill 1: 70 + 30 = 100

Skill 2: 30 + 70 = 100

Therefore 70 + 30 = 30 + 70

> **The same sum is got by adding in either order**
>
> **A + B = B + A**

2) Associative property

Addition of problems involving more than two numbers, any two numbers added first do not change the result.

Examples

Add **a)** 40 + 30 + 60

      Skill 1: (40 + 30) + 60 = 70 + 60 = 130

      Skill 2: (60 + 40) + 30 = 100 + 30 = 130

      Skill 3: (30 + 60) + 40 = 90 + 40 = 130

> **We can arrange three or more addends in any order and still get the same sum**
>
> **(A + B) + C = (A + C) + A = (B + C) + A**

Examples

1) Fill in the missing numbers

   a) 7 + 4 = ☐ + 7

   b) 4 + (5 + ☐) = (6 + 4) + 5

2) Find the value of a

   a) (a + 20) + 10 = (10 + 20) + 6

   b) 40 + (15 + a) = 15 + (40 + 10)

2) Subtraction

Subtraction means to take away a number from another.

Examples

1) Subtract 8,456,782 from 10,200,420

$$\begin{array}{r} 10{,}200{,}420 \\ -8{,}456{,}782 \\ \hline 1{,}743{,}638 \end{array}$$

Notice:

$$\begin{array}{r} 36 \quad \text{Minuend} \\ -12 \quad \text{Subtrahend} \\ \hline 24 \quad \text{Difference} \end{array}$$

2) Thomas' salary was 127,400 Frw. It was reduced by 49,680 Frw. How much does he get now?

$$
\begin{array}{r}
127{,}400 \text{ Frw} \\
-49{,}680 \text{ Frw} \\
\hline
77{,}720 \text{ } Frw
\end{array}
$$

Exercises

1) What is the difference between 624 415 and 35 897?

2) Subtract the following:

a) 6,000,101 – 4,999,011 =

b) 3,642,110 kg – 1,563,276 kg =

c) 8,621,143 trees from 9,132,423 trees =

3) Ingabire had a debt of 7,683,942 Frw. If she pays 5,839,678 Frw, how much debt is left?

4) The population of a country grew from 6 784 512 to 9 201 076 in two years. What was the population increase over this period of time?

5) 3,567,342 babies were born in a country in 2016. Of these, 1,593,599 babies were girls. Find the number of boys.

6) A truck carrying 2,560,000 litres of milk was in an accident. 1,756,950 litres were split. How much milk remained?

7) A farmer harvested 8,320,165 kg of maize. By the end of the month, he had sold 6,826,759 kg. How much kilograms are still in his store?

8) Subtract 34,763 from 82,124.

9) By how much is 367 015 greater than 346 929?

10) What should be added to 7 248 974 to get 8 124 610?

11) Complete:

a)
$$
\begin{array}{r}
5\ 3\ 4\ . \\
-\ 2\ .\ 7\ 9 \\
\hline
.\ 6\ .\ 7
\end{array}
$$

b)
$$
\begin{array}{r}
8\ 2\ .\ 6 \\
-\ .\ 4\ 8\ . \\
\hline
4\ .\ 5\ 9
\end{array}
$$

c)
$$
\begin{array}{r}
7\ .\ 3\ 2 \\
-\ 2\ 6\ .\ 4 \\
\hline
.\ 7\ 5\ .
\end{array}
$$

d) 84 – . = 28

e) . – 352 = 119

3) Multiplication

Multiplication is a process of adding a number to itself a particular number of times.

Examples: If we say **10 × 6** we mean **10 + 10 + 10 + 10 + 10 + 10** until we get six times.

Examples

1) Multiply: 25,467 × 289

$$\begin{array}{r} 25{,}467 \\ \times \quad 289 \\ \hline 229203 \\ + \, 203736 \\ 50934 \\ \hline 7359963 \end{array}$$

Notice:

$$\begin{array}{rl} 24 & \textit{Multiplicand} \\ \times \, 3 & \textit{Multiplier} \\ \hline 24 & \textit{Product} \end{array}$$

Casting out 9

2) Multiply and prove your answer: 56,738 × 472

$$\begin{array}{r} 56{,}738 \\ \times \quad 472 \\ \hline 113476 \\ + 397166 \\ 226952 \\ \hline 26780336 \end{array}$$

5+6+7+3+8=2+9=1+1=2

4+7+2=1+3=4

2+6+7+8+0+3+3+6=3+5=8

4×2=8
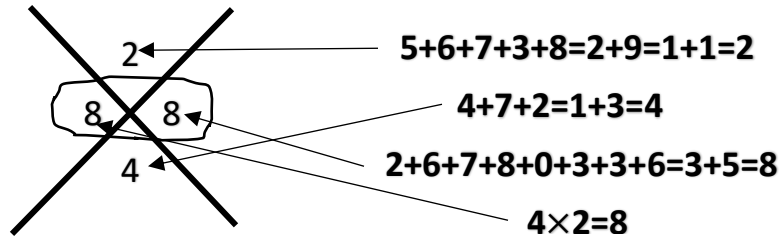
Exercises

1) When multiplying 408 by 23, Peter forgot to use 0 of 408. Find the total error he made.

2) A train carries 1 640 passengers a trip. How many passengers will it carry if it makes 15 trips?

3) One book cost 427 Frw. How much do 378 similar books cost?

4) A parade of soldiers was made up of 87 rows. There are 50 soldiers in each row. How many soldiers were there?

5) There are 24 bottles in a crate of Soda. How many bottles are there in 75 crates of Soda?

6) How many eggs are there in 59 trays of eggs?

7) How many books are there in 54 dozens of books?

8) A man multiplied 728 by 36 and forgot to use 8 of 728. Calculate the total error he made.

9) What is the product of 639 and 87?

10) Complete:

a) 35 × . = 840

b) . × 15 = 390

Properties of multiplication.

1) Commutative property

The result after multiplying two numbers in any order remains the same.

Examples: a) Multiply: 20 × 8

Skill 1: 20 × 8 = 160

Skill 2: 8 × 20 = 160

Therefore 20 × 8 = 8 × 20

b) Multiply: 15 × 4

Skill 1: 15 × 4 = 60

Skill 2: 4 × 15 = 60

> # The same product is got by multiplying two numbers in either order
>
> ## A ×B = B × A

2) Associative property

Multiplication problems involving more than two numbers, any two numbers multiplied first, the result remains the same.

Example: Multiply: 4 × 8 × 5

Skill 1: (4 × 8) × 5 = 32 × 5 = 160

Skill 2: (5 × 4) × 8 = 20 × 8 = 160

Skill 3: (8 × 5) × 4 = 40 × 4 = 160

> **To multiply three or more numbers, the first two numbers multiplied first, do not change the result. (A × B) × C = (A × C) × A = (B × C) × A**

3) Distributive property

To work out numbers using distributive property, distribute multiplication with numbers inside the brackets using addition and subtraction only

Examples: a) Calculate: 4(5 + 6)

4(5 + 6) = 4 x 11 = 44

b) Work out: 3 × 8 − 3 × 2

3 × 8 − 3 × 2 = 3(8 − 2) = 3 × 6 = 18

c) Calculate: 25 × 64 + 25 × 36

25 × 64 + 25 × 36 = 25(64 + 36)

=25 × 100 = 2 500

d) Evaluate: 65 × 1548 − 65 × 548

65 × 1548 − 65 × 548 = 65(1548 − 548)

= 65 x 1000 = 65,000

Exercise

1) Fill in the missing numbers

a) 39x(82+ ☐)=39x100

b) $76 \times (163 - \boxed{}) = 7,600$

2) Work out using distributive property.

a) $82 \times 726 + 82 \times 274$

b) $1836 \times 48 - 836 \times 48$

Quick multiplication

1. Quick multiplication by 10, 100, 1000, etc

To multiply any number by 10, 100, 1000, 10 000, 100 000 etc, simply add the zeros to the given number.

Examples

Calculate:

a) 25 x 10 = **250**

b) 386 x 1000 = **386,000**

c) 74 x 100 = **7,400**

d) 5 x 10 000 = **50 000**

2. Quick multiplication by 2

To multiply a number by 2, add the same number to itself.

Examples

Work out:

a) 243 x 2 =

$$\begin{array}{r} 243 \\ +243 \\ \hline 486 \end{array}$$

b) 2579 x 2

$$\begin{array}{r} 2579 \\ +2579 \\ \hline 5158 \end{array}$$

3) Quick multiplication by 5

To multiply a number by 5, multiply it by 10 and divide the result by 2

Examples

Effectuate: a) 275×5

$$275 \times 5 = (275 \times 10) \div 2$$
$$= 27500 \div 2 = 1\ 380$$

b) 789×5

$$789 \times 5 = (789 \times 10) \div 2$$

$$= 7890 \div 2$$
$$= 3\ 945$$

4. <u>Quick multiplication by 50</u>

To multiply a number by 50, multiply it by 100 and divide the result by 2.

<u>Examples</u>

Work out:

   a) 437×50

   $437 \times 50 = (437 \times 100) \div 2$
   $$= 43700 \div 2$$
   $$= 21\ 850$$

   a) 689×50

   $689 \times 50 = (689 \times 100) \div 2$
   $$= 68900 \div 2$$
   $$= 34\ 450$$

5. <u>Quick multiplication by 25</u>

To multiply a number by 25, multiply it by 100 and divide the result by 4.

<u>Examples</u>

Work out: a) 746×25

   $746 \times 25 = (746 \times 100) \div 4$
   $$= 74600 \div 4$$
   $$= 18\ 650$$

   b) 379×25

   $379 \times 25 = (379 \times 100) \div 4$
   $$= 37900 \div 4$$
   $$= 9\ 475$$

6. <u>Quick multiplication by 20</u>

To multiply a number by 20, multiply it by 2 and then by 10.

<u>Examples</u>

Calculate: a) 475×20

   $475 \times 20 = 475 \times 2 \times 10 = 950 \times 10$
   $$= 9\ 500$$

   b) 247×20

   $247 \times 20 = 247 \times 2 \times 10$
   $$= 494 \times 10 = 4\ 940$$

7. <u>Quick multiplication by 9</u>

To multiply a number by 9, multiply it by 10 and subtract the original number from the result.

<u>**Examples**</u>

Work out: a) 354 × 9

354 × 9 = 354 × (10 – 1)

=354 × 10 – 354 × 1

=3540 – 354

=3 186

b) 672 × 9

672 × 9 = 672 × (10 – 1)

=672 × 10 – 672 × 1

=6720 – 672

=6 048

8. <u>Quick multiplication by 99</u>

To multiply a number by 99, multiply it by 100 and subtract the original number from the result.

<u>**Examples**</u>

Work out: a) 396 × 99

396 × 99 = 396× (100 – 1)

=396 × 100 – 396 × 1

=39600 – 396

=39 204

b) 2 485 × 99

2 485 × 99 = 2 485 × (100 – 1)

=2 485 × 100 – 2 485 × 1

=2 48500 – 2 485

=246 015

9. <u>Quick multiplication by 19</u>

To multiply a number by 19, multiply it by 20 and subtract the original number from the result.

<u>**Examples**</u>

Calculate: a) 846 × 19

846 × 19 = 846 × (20 – 1)

= 846 × 20 – 846 × 1

$$= 16\ 920 - 646$$
$$= 16\ 074$$

b) 758×19

$$758 \times 19 = 758 \times (20 - 1)$$
$$= 758 \times 20 - 758 \times 1$$
$$= 15\ 160 - 758$$
$$= 14\ 402$$

10. Quick multiplication by 49

To multiply a number by 19, multiply it by 50 and subtract the original number from the result.

Examples

Calculate: a) 837×49

$$837 \times 49 = 837 \times (50 - 1)$$
$$= 837 \times 50 - 837 \times 1$$
$$= 41\ 850 - 837$$
$$= 41\ 013$$

b) 528×49

$$528 \times 49 = 528 \times (50 - 1)$$
$$= 528 \times 50 - 528 \times 1$$
$$= 26\ 400 - 528$$
$$= 25\ 872$$

11. Quick multiplication by 11

To multiply a number by 11, multiply it by 10 and add the original number to the result.

Examples

Calculate: a) 368×11

$$368 \times 11 = 368 \times (10 + 1)$$
$$= 368 \times 10 + 368 \times 1$$
$$= 3680 + 368$$
$$= 4\ 048$$

b) $5\ 849 \times 11$

$$5\ 849 \times 11 = 5\ 849 \times (10 + 1)$$
$$= 5\ 849 \times 10 + 5\ 849 \times 1$$
$$= 58\ 490 + 5\ 849$$

= 64 339

4) Division

1. Division without a remainder

Division is a process of finding out how many times one number is contained in another.

Examples

1) Divide: 19 248 ÷ 8 = **02 406**

```
      02 406
  8 ) 19248
    − 0
      19
    − 16
      32
    − 32
      004
    − 0
      48
      48
      00
```

2) Divide 208 830 by 6

```
      034605
  6 ) 208 830
    − 0
      20
    − 18
      28
    − 24
      048
    − 48
      003
      0
      30
      30
      00
```

3) Divide: 52 728 ÷ 13 = **4 056**

```
      4 056
  13 ) 52 728
     − 52
       007
     − 0
       72
     − 65
       78
     − 78
       00
```

4) Work out: 60 835 ÷ 23

```
       2645
  23 ) 60835
     − 46
       148
     − 138
       103
     − 92
       115
     − 115
       000
```

```
      9 ——— Quotient
  3 ) 27 ——— Dividend
      ↓
    Divisor
```

5) Calculate: 1382010 ÷ 35

```
        39 486
  35 ) 12 82010
     − 0
       138
     − 105
       332
     − 315
       170
     − 140
       301
       280
       210
       210
       00
```

6) Divide 988 880 by 235

```
         4 208
  235 ) 988 880
      − 940
        488
      − 470
        188
      − 0
        1880
      − 1880
        0
```

Exercises

Divide the numbers:

1) 8 744 480 ÷ 215 =

2) 3008488 ÷ 124 =

3) 4 575 244 ÷ 68 =

4) 54 964 ÷ 28 =

5) 12 700 314 ÷ 27 =

6) 88 831 ÷ 211 =

7) 162 828 ÷ 36 =

8) 33 088 120 ÷ 95 =

2. Division with a remainder

1) Divide 6 425 628 ÷ 24

```
        267734
   24 )6425628
      −48
       162
      −144
        185
       −168
        176
       − 168
          082
           72
          108
           96
           12      Remainder
```

2) Divide 3 486 728 by 213

```
          16374
   213 )3486728
       −213
        1356
       −1278
         787
        −629
         1582
        − 1491
           918
          −852
            66      Remainder
```

Word problems involving division

1) When X is divided by 62, the quotient is 12 and the remainder is 9. Find the value of X.

X = dp + r

= (62 × 12) + 9

= 744 + 9

= 953

2) The government released 36 450 000 Frw to 50 youth societies. How much did each get?

Total = 36 450 000 Frw

Number of societies = 50

Each got = 36 450 000 Frw ÷ 50 = 729, 000 Frw

Exercises

1) A total of 54 142 books were distributed to 23 classes. How many books did each class get?

2) Share 2,026,800 Frw among 24 employees.

3) A soda bottling company packed 8,462,376 bottles of soda in crates each containing 24 bottles. Find the number of crates that were packed.

4) A sugar factory manufactured 12,960,648 kg of sugar in a year. How many kg of sugar were produced every month if the factory produces equal amounts of sugar monthly?

5) The electricity board put the lamp posts at a distance of 484 dm apart in the distribution of power to all roads in the city. How many posts were put along a street covering a length of 5 246 560 dm?

Divisibility test for numbers

Divisibility test refers to the shortest possible process through which to determine whether or not a given number can be divided by another without a remainder.

1) Divisibility test for 2

A number I divisible by 2 if it ends with an **even number.**

Examples

1) Is 46 734 divisible by 2?

Yes, it is divisible by 2 because the last digit is an even number

2) Among the following numbers which ones are divisible by 3?

67 094 , 22 229 , 400 001 and 17 930

Solution: 67 094 and 17 930 are divisible by 2

2) Divisibility test for 3

A number is divisible by 3 if the sum of its digits is divisible by 3.

Examples

1) Find out whether 5,241 is divisible by 3.

Sum of digits = 5 + 2 + 4 + 1 = 12

It is divisible by 3

2) Is 57,068 divisible by 3? Show how you reach the answer.

Sum of digits = 5 + 7 + 0 + 6 + 8 = 26

It is not divisible by 3

3) Divisibility test for 6

A number is divisible by 6 if it is divisible by both 2 and 3.

Examples

1) Find out if 93,546 is divisible by 6.

- **It is divisible by 2 because it ends with an even number.**
- **Sum of digits = 9 + 3 + 5 + 4 + 6 = 27**
  It is divisible by 3
- **Therefore, it is divisible by 6**

2) Without using a long calculation show if 70,314 is divisible by 6.

- **It is divisible by 2 because it ends with an even number.**
- **Sum of digits = 7 + 0 + 3 +1 + 4 = 15**
  It is divisible by 3
- **Therefore, it is divisible by 6**

4) Divisibility test for 4

A number is divisible by 4 if the last two digits are divisible by 4.

Examples

1) Without dividing show whether or not 295,672 is divisible by 4.

The last two digits are 72 and 72÷4 = 18

 It is divisible by 4

2) Among the numbers below which ones are not divisible by 4?

44 449 ; 71 536 , 80 802 and 53 952

Solution: 44 449 and 80 802 are not divisible by 4.

5) Divisibility test for 12

A number is divisible by 12 if it is divisible by both 3 and 4.

Examples

1) Find out if 11,148 is divisible by 12.

- **Sum of digits: 1 + 1 + 1 + 4 + 8 = 15**
  It is divisible by 3
- **The last two digits are 48 and 48÷4 = 12**
  It is divisible by 4
- **Therefore, it is divisible by 12**

2) Check whether 315,936 is divisible by 12.

- **Sum of digits: 3 + 1 + 5 + 9 + 3 + 6 = 27**
  It is divisible by 3
- **The last two digits are 36 and 36÷4 = 9**
  It is divisible by 4
- **Therefore, it is divisible by 12**

6) Divisibility test for 5

A number is divisible by 5 if it ends with 0 or 5.

Examples

Among the following numbers, which ones are divisible by 5?

55 558 ; 719 420 ; 400 004 ; 39 175

Solution: 719 420 and 39 175 are divisible by 5

7) Divisibility test for 8

A number is divisible by 8 if the last three digits are divisible by 8.

Examples

From the numbers below, choose those that are nit divisible by 8.

8 047 288 ; 715 000 ; 48 480 024 and 88 884

Solution: 8 047 288 and 88 884 are not divisible by 8.

8) Divisibility test for 9

A number is divisible by 9 if the sum of its digits is 9.

Examples

1) Is 12,222 divisible by 9? Show why.

Sum of digits = 1 + 2 + 2 + 2 + 2 = 9

It is divisible by 9

2) By not dividing, show if 84,753 is divisible by 9.

Sum of digits = 8 + 4 + 7 + 5 + 3 = 27

It is divisible by 9

9) Divisibility test for 10

A number is divisible by 10 if it ends with 0.

Examples

Circle the numbers that are divisible by 10.

576 930 ; 1 010 101 ; 79 360 and 80 005

10) Divisibility test for 11

A number is divisible by 11 if the difference between the sum of odd position numbers and even position numbers is 0 or divisible by 11.

Examples

1) Is 383,482 divisible by 11?

3*83*,48*2

- **Sum 1 = 3 + 3 + 8 = 14**
- **Sum 2 = 8 + 4 + 2 = 14**

- **Difference = 14 – 14 = 0**
- **It is divisible by 11**

2) Find out if 928 092 divisible by 11.

$$9^*28^*,09^*2$$

- **Sum 1 = 9 + 8 + 9 = 26**
- **Sum 2 = 2 + 0 + 2 = 4**
- **Difference = 26 – 4 = 22**
- **It is divisible by 11**

11) <u>Divisibility test for 7</u>

A number is divisible by 7 if continuous subtraction of twice the last digits from the remaining number gives a number that is divisible by 7.

<u>Examples</u>

Without dividing show if 26 894 is divisible by 7.

26 89<u>4</u>	26 89	268	26
<u>× 2</u>	<u> − 8 </u>	<u> − 2 </u>	<u>−12</u>
8	268<u>1</u>	26<u>6</u>	14
	<u>× 2</u>	<u>× 2</u>	
	2	12	

It is divisible by 7

12) <u>Divisibility test for 13</u>

A number is divisible by 7 if continuous addition of four times the last digits to the remaining number gives a number that is divisible by 13.

<u>Examples</u>

Without dividing show if 4 537 is divisible by 13.

453<u>7</u>	453	48	5
<u>× 4</u>	<u>+ 28 </u>	<u>+ 4 </u>	<u>+ 8</u>
28	48<u>1</u>	5<u>2</u>	13
	<u>× 4</u>	<u>× 4</u>	
	4	8	

TOPIC 5: <u>SEQUENCE AND PATTERNS</u>

A sequence or progression is a series of numbers which have a particular order.

<u>TYPES OF SEQUENCES</u>

1) **Increasing progression**

When the numbers in a sequence are in increasing order, it is either addition or multiplication.

Examples

Fill in the missing numbers:

1) 3 , 6 , 12 , **15 , 18 , 21**

    +3 +3 +3   +3   +3

2) 2 , 4 , 8 , 16 , **32 , 64 , 128**

   ×2 ×2 ×2   ×2   ×2   ×2

3) 20 , 30 , 40 , 50 , **60 , 70 , 80**

   +10   +10 +10 +10 +10 +10

4) 1 , 3 , 9 , 27 , **81 , 273**

   ×3 ×3 ×3   ×3   ×3

5) 500 , 700 , 900 , **1100 , 1300 , 1500**

    +200   +200   +200   +200   +200

More about increasing sequence

Fill in the next two missing numbers

1) 3 , 4 , 7 , 12 , **19 , 26**

    +1 +3   +5   +7   +9 (Adding odd numbers)

2) 2 , 4 , 12 , 48 , **240 , 1440**

   ×2 ×3 ×4   ×5   ×6 (Multiplying by whole numbers)

3) 4 , 6 , 9 , 13 , **20 , 31**

    +2 +3   +5   +7   +11 (Adding prime numbers)

4) 6 , 6 , 8 , 12 , **18 , 26**

    +0   +2   +4   +6   +8 (Adding even numbers)

5) 5 , 10 , 17 , 26 , **37 , 50**

     +5   +7   +9   +11   +13 (Adding odd numbers from 5)

6) 12 , 22 , 34 , 48 , **64 , 80**

    +10   +12   +14   +16   +16 (Adding even number from 10)

7) 4 , 8 , 32 , 192 , **1 536 , 15 360**

    ×2   ×4   ×6   ×8   ×10 (Multiplying by even numbers)

8) 3 , 3 , 9 , 45 , **315 , 2835**

    ×1   ×3   ×5   ×7   ×9 (Multiplying by odd numbers)

9) 2 , 4 , 12 , 60 , **420 , 4620**

    ×2   ×3   ×5   ×7   ×11 (Multiplying by prime numbers)

Increasing sequences including fractions

Fill in the next two missing numbers

1) 2 ; $2\frac{1}{2}$; 3 ; $3\frac{1}{2}$; $\mathbf{4}$; $\mathbf{4\frac{1}{2}}$

Adding $\frac{1}{2}$

2) $\frac{1}{2}$; $\frac{2}{3}$; $\frac{3}{4}$; $\frac{5}{6}$; $\mathbf{\frac{6}{7}}$; $\mathbf{\frac{7}{8}}$

Adding 1 to both numerator and denominator

3) 1 ; $1\frac{1}{2}$; $1\frac{1}{3}$; $1\frac{1}{4}$; $\mathbf{1\frac{1}{5}}$; $\mathbf{1\frac{1}{6}}$

$+\frac{1}{2}$ $\quad +\frac{1}{3}$ $\quad +\frac{1}{4}$ $\quad +\frac{1}{5}$ $\quad +\frac{1}{6}$

4) $\frac{1}{3}$; $\frac{1}{6}$; $\frac{1}{12}$; $\frac{1}{24}$; $\mathbf{\frac{1}{48}}$; $\mathbf{\frac{1}{96}}$

Multiplying by $\frac{1}{2}$

5) 3 ; $\frac{3}{4}$; $\frac{3}{16}$; $\mathbf{\frac{3}{64}}$; $\mathbf{\frac{3}{256}}$

Multiplying by $\frac{1}{4}$

6) 15 ; $16\frac{1}{2}$; 18 ; $19\frac{1}{2}$; $\mathbf{21}$; $\mathbf{22\frac{1}{2}}$

Adding $1\frac{1}{2}$

Increasing sequences including decimals

Find the next numbers in the sequences

1) 5 ; 5.5 ; 6 ; 6.5 ; $\mathbf{7}$; $\mathbf{7.5}$; $\mathbf{8}$

Adding 0.5

2) 70 ; 73.5 ; 77 ; 80.5 ; $\mathbf{84}$; $\mathbf{87.5}$

Adding 3.5

3) 0.2 ; 0.8 ; 1.4 ; 2 ; 2.6 ; $\mathbf{3.2}$; $\mathbf{3.8}$

Adding 0.6

4) 8.3 ; 8.6 ; 8.9 ; $\mathbf{9.2}$; $\mathbf{9.5}$

Adding 0.3

2) Decreasing progression

Fill in the missing numbers:

1) 80 , 70 , 60 , $\mathbf{50}$, $\mathbf{40}$, $\mathbf{30}$

$\quad$ -10 $\quad$ -10 $\quad$ -10 $\quad$ -10 $\quad$ -10

2) 32 , 16 , 8 , **4 , 2 , 1**

   ÷2   ÷ 2 ÷ 2 ÷ 2 ÷2

3) 68 , 61 , 54 , 47 , **40 , 33 , 26**

   -7   -7   -7   -7   -7   -7

4) 243 , 81 , 27 , **9 , 3 , 1**

   Dividing by 3

Sequences including integers

Write down the next numbers in the sequence

1) 4 ; -8 ; 16 ; -32 ; 64 ; -128

   Multiplying by -2

2) 16 ; 12 ; 8 ; **4 ; 0 ; -4 ; -8**

    -4   -4   -4   -4   -4   -4

3) -17 ; -12 ; -7 ; **-2 ; +3 ; +8**

   Adding +5

4) -3 ; -7 ; -11 ; -15 ; **-19 ; -23**

    -4   -4   -4   -4   -4

Some specific sequences

Fill in the next missing numbers

1) 2 ; 5 ; 11 ; 23 ; **67 ; 133**

   ×2 +1 (Multiply by 2 and add 1)

2) 3 ; 5 ; 9 ; 17 ; **33 ; 65**

   ×2 -1 (Multiply by 2 and subtract 1)

3) 1 ; 4 ; 9 ; 16 ; 25 ; **36 ; 49**

   Square numbers

4) 1 ; 8 ; 27 ; 64 ; **125 ; 216**

   Cube numbers

5) Complete the tables below

5	**9**	8	6
26	82	65	**37**

   Square numbers plus one

48	**15**	99	143

7	4	10	**12**

Square numbers minus one

Exercises

Find the next numbers in the sequences

1) 36 ; 6 ; 1 ; …….. ; ………
2) $\frac{1}{8}$; $\frac{1}{4}$; $\frac{1}{2}$; ………. ; ……….
3) 1 ; -3 ; 9 ; -27 ; ……… ; ……..

Complete the table below:

1	.	4	8
1	0.5	.	0.125

-7	-16	.	12
-5	.	0	14

TOPIC 6: TYPES OF NUMBERS

1. Whole numbers

Whole numbers are numbers with no fraction and begins with zero. Whole numbers tell us **how many** members a group has.

Eg: {0, 1, 2, 3, 4, 5, 6, 7, …………}

2. Counting numbers

Counting numbers are numbers used while counting the number of members a group has.

Eg: {1, 2, 3, 4, 5,…………….}

Notice: Zero is not a counting number.

3. Even numbers

Even numbers refer to the numbers which are divisible by 2.

Eg: {0, 2, 4, 6, 8, 10, 12,……………….}

4. Odd numbers

Odd numbers refer to the numbers which are not divisible by 2.

Eg: {1, 3, 5, 7, 9, 11, 13, 15, 17, 19,………………….}

5. Ordinal numbers

Ordinal numbers are numbers that tell us the positions of members of a group

Eg: 1ˢᵗ, 2ⁿᵈ, 3ʳᵈ, 4ᵗʰ, 5ᵗʰ, ……..

6. Consecutive numbers

Consecutive numbers refer to numbers which follow one another in series.

There are four types of consecutive numbers:

i) Consecutive counting numbers eg: 1, 2, 3, 4, 5, 6, 7, 8,……………..

ii) Consecutive even numbers eg: 0, 2, 4, 6, 8, 10,…………………….

iii) Consecutive odd numbers eg: 1, 3, 5, 7, 9, 11, ……………………..

iv) Consecutive prime numbers eg: 2, 3, 5, 7, 11, 13, 17, 19, ………….

7. Square numbers

Square numbers are numbers got by multiplying a number by itself.

Eg: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 225, …………………..

a) Finding the square of numbers

To get the square of any number, multiply it by itself.

Examples

- The square of 1 is 1^2 = **1 x 1 = 1**
- The square of 2 is 2^2 = **2 x 2 = 4**
- The square of 3 is 3^3 = **3 x 3 = 9**
- The square of 12 is 12^2 = **12 x 12 = 144**
- The square of 20 is 20^2 = **20 x 20 = 400**
- The square of 150 is 150^2 = **150 x 150 = 22 500**

b) Square of numbers ending in 5

To get the square of numbers ending in 5, write immediately 25 at the end and multiply the remaining number in front of 5 by the number that follows if in whole numbers.

Examples

Find the square of the following numbers:

a) Square of 15 = 225

      1 x 2 = 2

b) Square of 45 = 20 25

      4 x 5 = 20

c) Square of 75 = 5625

      7 x 8 = 56

d) The square of 10 = 11025

10 x 11 = 110 ⤵

c) Square root

➤ The square root of any number is a number which was multiplied by itself to get that number.

➤ The symbol of square root is $\sqrt{}$

Examples

1) Calculate the square root of 64

$$
\begin{array}{c|c}
2 & 64 \\
2 & 32 \\
2 & 16 \\
2 & 8 \\
2 & 4 \\
2 & 2 \\
& 1
\end{array}
$$

Then $\sqrt{64}$ = 2 x 2 x 2 = 8

2) Find the number that was multiplied by itself to get 144.

$$
\begin{array}{c|c}
2 & 144 \\
2 & 72 \\
2 & 36 \\
2 & 18 \\
3 & 9 \\
3 & 3 \\
& 1
\end{array}
$$

$\sqrt{144}$ = 2 x 2 x 3 = 12

Exercises

1) Find the square root of 625

2) Find the number that is multiplied by itself to be 81

3) Calculate: $\dfrac{\sqrt{169} + \sqrt{225}}{2}$

4) Work out: $\dfrac{\sqrt{100} + \sqrt{256}}{\sqrt{4}}$

5) Find: $\dfrac{\sqrt{289} - \sqrt{121}}{\sqrt{25}}$

8. Cube numbers

Cube numbers are numbers got when a natural number is multiplied by itself two times.

Eg: 1, 8, 27, 64, 125, 216, 343, 512, …………

a) Finding the cube of numbers

To get the cube of any number, multiply it by itself three times.

Examples

- The cube of 4 is 4^3 = **4 x 4 x 4 = 64**
- The cube of 10 is 10^3 = **10 x 10 x 10 = 1000**
- The cube of 20 is 20^3 = **20 x 20 x 20 = 8000**
- The cube of 4 is 4^3 = **4 x 4 x 4 = 64**
- The cube of 15 is 15^3 = **15 x 15 x 15 = 3375**

b) Finding the cube root

➢ The cube root of a number is a number which was multiplied by itself two times.

➢ The symbol of cube root is $\sqrt[3]{}$

Examples

1) Find the cube root of 512

```
2 | 512
2 | 256
2 | 128   Then ∛512 = 2 x 2 x 2 = 8
2 | 64
2 | 32
2 | 16
2 | 8
2 | 4
2 | 2
  | 1
```

2) Calculate the cube root of 216

```
2 | 216
2 | 108
2 | 54    Then ∛216 = 2 x 3 = 6
3 | 27
3 | 9
3 | 3
```

Exercises

1) Find the cube root of 343

2) Find the number that was multiplied by itself two times to be 729.

3) Calculate $\sqrt[3]{125}$ - $\sqrt[3]{64}$

4) Evaluate: $\dfrac{\sqrt[3]{1728} + \sqrt[3]{1000}}{\sqrt[3]{64}}$

MORE ABOUT SQUARE ROOT

a) Properties of square root

1) Product property of square root

The product of the square root of two numbers is equal to the square root of the poduct of the two numbers.

Examples

1) Work out: $\sqrt{16} \times \sqrt{4}$

$$\mathbf{\sqrt{16} \times \sqrt{4} = \sqrt{16 \times 4}}$$
$$\mathbf{= \sqrt{64}}$$
$$\mathbf{= 8}$$

2) Evaluate: $\sqrt{18} \times \sqrt{8}$

$$\mathbf{\sqrt{18} \times \sqrt{8} = \sqrt{18 \times 8}}$$
$$\mathbf{= \sqrt{144}}$$
$$\mathbf{= 12}$$

2) Quotient property of square root

The square root of a fraction is equal to the square root of the numerator divided by the square root of the denominator.

Examples

1) Evaluate: $\sqrt{\dfrac{64}{144}}$

$$\mathbf{\sqrt{\dfrac{64}{144}} = \dfrac{\sqrt{64}}{\sqrt{144}}}$$
$$\mathbf{= \dfrac{8}{12}}$$
$$\mathbf{= \dfrac{2}{3}}$$

2) Calculate: $\sqrt{\dfrac{72}{18}}$

$$\sqrt{\frac{225}{25}} = \frac{\sqrt{225}}{\sqrt{25}}$$
$$= \frac{15}{5}$$
$$= 3$$

3) Square root of a decimal number

To calculate the square root of a decimal number, first change the decimal number into a fraction.

Examples

1) What is the square root of 0.81

$$\sqrt{0.81} = \sqrt{\frac{81}{100}} = \frac{\sqrt{81}}{\sqrt{100}}$$
$$= \frac{9}{10}$$
$$= 0.9$$

2) Evaluate: $\sqrt{2.56}$

$$\sqrt{2.56} = \sqrt{\frac{256}{100}} = \frac{\sqrt{256}}{\sqrt{100}}$$
$$= \frac{16}{10}$$
$$= 1.6$$

9. Prime numbers

Prime numbers refer to numbers which have only two factors, **one** and **itself**.
Eg: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, ……………

Notice: *The smallest prime number is 2

*The only one prime number which is even number is 2

Exercises

1) What is the sum of the prime numbers between o and 20?
2) Find the sum of the prime numbers between 20 and 30
3) Calculate the average of the prime numbers between 20 and 40.
4) Find the mean of the prime numbers between 10 and 20

TOPIC 5: PRIME FACTORISATION

Prime factorisation is a way of finding the prime factors of a number.
When prime factorising:

1) We only use prime numbers
2) We always start from the lowest prime number to the largest

Examples

1) Prime factorise 96

```
2 | 96
2 | 48
2 | 24
2 | 12
2 | 6
3 | 3
  | 1
```

2) Express 252 as a product of its prime factors

```
2 | 252
2 | 126
3 | 63
3 | 21
7 | 7
  | 1
```

$252 = 2 \times 2 \times 3 \times 3 \times 7$

Exercises

1) Express 216 as a product of its prime factors.
2) Prime factorise 336.
3) Express 2520 as a product of its prime factors.
4) Express 420 as a product of its prime factors.
5) Prime factorise 1980 and write it as a product of its prime factors.
6) Express 910 as a product of its prime factors.

Finding the prime factorised number

1) Find the number which has been prime factorised to get 2 x 2 x 2 x 3 x 5

2 x 2 x 2 x 3 x 5 = 120

The number is 120

2) Which number is factorised to get 2^2 x 3^2 x 5?

2^2 x 3^2 x 5 = 2 x 2 x 3 x 3 x 5

= 180

The number is 180

3) Find the number whose factorization s 5 x 5 x 7 x 17

5 x 5 x 7 x 17 = 2 975

The number is 2 975

Finding the unknown prime factor

1) The prime factors of 60 are 2 x 2 x p x 5. Find the value of p.

```
2 | 60        2 x 2 x p x 5
```

2	30
3	15
3	5
	1

or p = 60 ÷ (2 x 2 x 5)

= 60 ÷ 20

= 3

Therefore, p = 3

2) The prime factors of 90 are 2 x 3 x 3 x n. Find the value of n.

n = 90 ÷ (2 x 3 x 3)

= 90 ÷ 18

= 5

MULTIPLES AND FACTORS OF NUMBERS

1) Multiples of numbers

*A multiple is a number that contains another number the exact number of times.

*To get the multiples of any number, multiply it by counting numbers.

Examples

➢ The multiples of 2 are = {2, 4, 6, 8, 10, 12, 14, 16, 18, ………..}

➢ The multiples of 5 are = {5, 10, 15, 20, 25, 30, 35, ……………….}

➢ The multiples of 8 are = {8, 16, 24, 32, 40, 48, 56, ………………}

➢ The multiples of 12 are = {12, 24, 36, 48, 60, 72, ………………}

a) More about multiples

1) Write down the multiples of 5 between 20 and 40.

={25, 30, 35}

2) List down the multiples of 6 less than 40.

={6, 12, 18, 24, 30, 36}

3) Write the first six multiples of 7.

={7, 14, 21, 28, 35, 42}

4) How many multiples of 9 are there between 20 and 60?

={27, 36, 45, 54}

There are 4 multiples

5) What is the sum of multiples of 5 between 10 and 30?

={15, 20, 25}

Sum = 15 + 20 + 25

= 60

6) Find the sum of the multiples of 2 less than 10.

>={2, 4, 6, 8}
>
>Sum = 2 + 4 + 6 + 8
>
>= 20

b) Common multiples

1) List the common multiples of 4 and 6 less than 50.

- **Multiple of 4 = {4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48}**
- **Multiples of 6 = {6, 12, 18, 30, 36, 42, 48}**
- **Common factors = {12, 24, 36, 48}**

2) Find the common multiples of 8 and 12 less than 80

- **Multiples of 8 = {8, 16, 24, 32, 40, 48, 56, 64, 72}**
- **Multiples of 12 = {12, 24, 36, 48, 60, 72}**
- **Common multiples = {24, 48, 72}**

3) What is the Lowest Common Multiple of 10 and 15?

- **Multiples of 10 = { 10, 20, 30, 40, 50, 60, 70, 80, …….}**
- **Multiples of 15 ={15, 30, 45, 60, 75, 90, ………}**
- **Lowest Common Multiple is 30**

4) Find the LCM of 20 and 30.

- **Multiples of 20 = { 20, 40, 60, 80, …….}**
- **Multiples of 30 ={30, 60, 90, 120, ……..}**
- **Lowest Common Multiple is 60**

2) Factors or Divisors

A factor or a divisor is a number that divides another exactly (without a remainder).

Notice: * 1 is a factor of every number.

* Any number is a factor of itself.

Examples

1) List down all factors of 12

>**1 x 12**
>
>**2 x 6**
>
>**3 x 4**
>
>**Factors of 12 ={1, 2, 3, 4, 6, 12}**

2) What are the divisors of 20?

>**1 x 20**

2 x 10

4 x 5

Factors of 12 ={1, 2, 4, 10, 20}

3) Write down all factors of 50.

1 x 50

2 x 25

5 x 10

Factors of 12 ={1, 2, 5, 10, 25, 50}

4) How many factors does 40 have?

1 x 40

2 x 20

4 x 10

5 x 8

Factors of 12 ={1, 2, 4, 5, 8, 10, 20, 40}

It has 8 factors

a) More about factors

1) What is the sum of factors of 8?

Factors of 8 ={1, 2, 4, 8}

Sum =1 + 2 + 4 + 8

=15

2) Find the sum of the factors of 15

Factors of 15 ={1, 3, 5, 15}

Sum =1 + 3 + 5 + 15

=24

b) Common factors or divisors

1) Find the common divisors of 16 and 20.

- **Factors of 16 ={1, 2, 4, 8, 16}**
- **Factors of 20 ={1, 2, 4, 5, 10, 20}**
- **Common factors ={1, 2, 4}**

2) What are the factors of 24 and 36?

- **Factors of 24 ={1, 2, 3, 4, 6, 8, 12, 24}**
- **Factors of 36 ={1, 2, 3, 4, 6, 9, 12, 18, 36}**
- **Common factors ={1, 2, 3, 4, 6, 12}**

3) What is the greatest common factor of 30 and 45?

- **Factors of 30 ={1, 2, 3, 5, 6, 10, 15, 30}**

- **Factors of 45 ={1, 3, 5, 9, 15, 45}**
- **Greatest Common Factors is 15**

4) Find the Greatest Common Factor of 40 and 60.

- **Factors of 40 ={1, 2, 4, 5, 10, 20, 40}**
- **Factors of 60 ={1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60}**
- **Greatest Common Factors is 20**

3) Lowest Common Multiple and Greatest Common factor

1) Find the LCM and GCF of 20 and 30

2*	20	30
2	10	15
3	5	15
5*	5	5
	1	1

LCM = 2 x 2 x 3 x 5 = 60

GCF = 2 x 5 = 10

2) What is the LCM and GCF of 60, 80 and 120?

2*	60	80	120
2*	30	40	60
2	15	20	30
2	15	10	15
3	15	5	15
5*	5	5	5
	1	1	1

LCM = 2 x 2 x 2 x 2 x 3 x 5 = 240

GCF = 2 x 2 x 5 = 20

Exercises

1) Find the lowest common multiple of 24, 36 and 48.

2) What is the greatest common factor of 75 and 135?

3) Find the LCM and GCF of 40, 60 and 80.

4) Calculate the LCM and GCF of 45, 75 and 105.

5) What is the highest common divisor of 120 and 160?

More about LCM and GCF

1) The LCM and GCF of two numbers are 120 and 20 respectively. If one number is 40, find the second number.

LCM = 120

$$GCF = 20$$

$$1^{st} \text{ number} = 40$$

$$2^{nd} \text{ number} = \frac{LCM \times GCF}{1^{st}number} = \frac{120 \times 20}{40} = \frac{240}{40} = 60$$

2) The LCM and GCF of two numbers are 240 and 40 respectively. If one number is 120, find the second number.

$$LCM = 240$$

$$GCF = 40$$

$$1^{st} \text{ number} = 120$$

$$2^{nd} \text{ number} = \frac{LCM \times GCF}{1^{st}number} = \frac{240 \times 40}{120} = 80$$

3) Calculate the sum of LCM and GCF of 24, 36 and 48.

2*	24	36	48
2*	12	18	24
2	6	9	12
2	3	9	6
3*	3	9	3
3	1	3	1
		1	

LCM = 2 x 2 x 2 x 2 x 3 x 3 = 144

GCF = 2 x 2 x 3 = 12

Sum = 144 + 12 = 156

4) The GCF of two numbers is 48 and one of the two numbers is 24. Find the second number.

$$GCF = 48$$

$$1^{st}number = 24$$

$$2^{nd}number = GCF + 1^{st}number = 48 + 24 = 72$$

Exercises

1) Find the difference between the LCM and GCF of 60 and 90.

2) What is the quotient of the LCM and GCF of 100 and 150?

3) Find the product of the lowest common multiple and the greatest common factor of 400 and 600.

4) The GCF of two numbers is 80 and one of the two numbers is 240. Find the second number.

More about GCF

1) Find the great number that can divide 24, 36 and 48.

GCF of 24, 36 and 48

2*	24	36	48
2*	12	18	24
2	6	9	12
2	3	9	6
3*	3	9	3
3	1	3	1
		1	

GCF = 2 x 2 x 3 = 12

The number is 12

2) Joel fetched 60 litres of water in the morning and 72 litres in the evening. Find the capacity of the biggest container Joel used in both instances.

3) What is the greatest number that can divide 36 and 54 without leaving a remainder?

4) Four wooden poles have lengths 280cm, 336 cm, 476 cm and 420 cm. the owner wishes to cut them into shorter pieces of equal length. Find the greatest possible length of each piece if no wood is left over.

5) Musa collected 48 kg of okra seeds from on garden and 84 kg from another. Find the mass of the pack that can be used in both instances without leaving any okra seeds in the garden.

6) Juma buys animal feeds in sacks of 90 kg, 70 kg, 50 kg and 40 kg. He sells these feeds in small quantities. What is the largest quantity of feed that can be measured out from these sacks without any feed being left over?

7) Find the greatest number that can divide 30, 45 and 60.

More about LCM

1) Three bells ring at intervals of 20, 30 and 40 minutes respectively. If they are started at the same time, after how long will they ring together again?

LCM of 20, 30 and 40

2*	20	30	45
2	10	15	45
3	5	15	45
3	5	15	15

```
5*  5   5   5
    1   1   1
```

LCM = 2 x 2 x 3 x 3 x 5 = 180

They will ring together after 180 minutes = 3hrs

2) A number is such that when it is divided by 8, 12 or 16 the remainder is always 5. Find the number.

2*	8	12	16
2*	4	6	8
2	2	3	4
2	1	3	2
3		3	1
		1	

LCM = 2 x 2 x 2 x 2 x 3= 48

The number is LCM + remainder = 48 + 5 = 53

3) Three flashes flash at intervals of 40, 60 and 80 seconds respectively. If they have been started at once, how soon after will they flash together again?

4) Two bells ring at intervals of 15 and 20 minutes.
   a) After how long will the bells ring at the same time?
   b) If they both ring at 9:00 am, at what time will they ring together again?

5) Three flashes flash at intervals of 6 hours, 10 hours and 15 hours respectively. How long will it take before they ring together again?

6) Thee taxis leave the park at intervals of 15, 20 and 25 minutes. After how long will the taxis leave the park at the same time?

7) Three buses arrive at a bus park at intervals of 30, 40 and 45 minutes respectively. How long will the buses take to arrive at the park at the same time if their first arrival time was the same?

8) Find the least number that can be divided by 12, 24 and 36 without leaving a remainder?

9) What number when divided by 12, 18 and 20 leaves no remainder?

10) A number is such that when it is divided by 15, 20 or 30 the remainder is always 3. Find that number.

TOPIC 6: <u>UNEQUAL SHARES</u>

1) <u>When sum and difference are given</u>

1) Mary and Peter shared 5,000 Frw such that Mary got 1,000 Frw as much as Peter. How much money did each get?

Sum = 5,000 F

Difference = 1,000 F

Peter $= \frac{sum-difference}{2} = \frac{5,000\ F-1,000\ F}{2} = \frac{4,000\ F}{2} =$ **2,000 Frw**

Peter $= \frac{sum+difference}{2} = \frac{5,000\ F+1,000\ F}{2} = \frac{6,000\ F}{2} =$ **3,000 Frw**

2) A teacher is 24 years older than his pupil. The sum of their ages is 66 years. How old is each?

Sum = 66 years

Difference = 24 years

Pupil $= \frac{sum-difference}{2} = \frac{66-24}{2} = \frac{42}{2} =$ **21 years**

Teacher $= \frac{sum+difference}{2} = \frac{66+24}{2} = \frac{90}{2} =$ **45 years**

3) The sum of two numbers is 48 and their difference is 12. Find the numbers.

4) Angelique is 7 years younger than John. How old is each if their total age is 37 years?

5) In primary five, the first pupil scored 38 marks more than the last. The sum of the marks for the two is was 132. How many marks did each get?

6) A table costs 2,000 frw more than a chair. If the total cost is 14,000 frw, how much did each cost?

7) Share 37 mangoes between Mutesi and Rurangwa such that Rurangwa gets 9 mangoes more than Mutesi.

8) There are 40 more boys than girls in a class. If there are 140 pupils, altogether, how many boys and girls are there?

9) A book costs 3oo frw more than a pen. If their total cost is 900 frw, find the cost of each.

10) The length of the rectangle is 10 cm more than the width. If the perimeter of the rectangle is 50 cm, find:

   a) The width of the rectangle.

   b) The length of the rectangle.

   c) The area of the rectangle.

2) When sum and quotient are given

1) Sano is thrice as old as James and their total age is 60 years. How old is each?

Sum = 60 years

Q = 3

James = sum ÷ (Q + 1)

$\qquad$ **= 60 ÷ (3 + 1)**

$$= 60 \div 4$$
$$= 15$$

Sano = 15 x Q
$$= 15 \times 3$$
$$= 45$$

2) The sum of two numbers is 240 and their quotient is 7. What are the two numbers?

Sum = 240

Q = 7

James = sum ÷ (Q + 1)
$$= 240 \div (7 + 1)$$
$$= 240 \div 8$$
$$= 30$$

Sano = 30 x Q
$$= 30 \times 7$$
$$= 210$$

3) Divide 100,000 frw between Helen and Felix such that the quotient of their shares is 4.

4) In a school of 501 pupils, the number of girls is twice the number of boys. Find:
   a) The number of boys.
   b) The number of girls.

5) Karemera's salary is twice as much as his wife's salary. If both earn 500,000Frw, calculate the salary of each.

6) A book and a pen cost 67,200 Frw. A book costs 5 tmes more than a pen. How much is a pen?

7) The sum of two numbers is 90 and their quotient is 5. Find the numbers.

8) In a class of 45 pupils, the quotient of girls to boys is 2. How many boys and girls are there?

9) Divide 450 in two such that their quotient is 9.

10) Betty is four times as old as Nganji and the sum of their ages is 80 years. How old is each?

3) When difference and quotient are given

1) There are 240 more girls than boys at Susa primary school. If the number of girls is thrice as many as that of boys, how many boys and girls are there?

Difference = 240 years

Q = 3

$$\textbf{Boys} = \textbf{diff} \div (\textbf{Q} - \textbf{1})$$
$$= 240 \div (3 - 1)$$
$$= 240 \div 2$$
$$= 120 \textbf{ boys}$$
$$\textbf{Girls} = 120 \times \textbf{Q}$$
$$= 120 \times 3$$
$$= 360 \textbf{ girls}$$

2) The difference between two numbers is 80 and their quotient is 5. Fin the numbers.

$$\textbf{Difference} = 80$$
$$\textbf{Q} = 5$$
$$2^{nd}\textbf{number} = \textbf{diff} \div (\textbf{Q} - \textbf{1})$$
$$= 80 \div (5 - 1)$$
$$= 80 \div 4$$
$$= 20$$
$$1^{st}\textbf{numbe} = 120 \times \textbf{Q}$$
$$= 120 \times 3$$
$$= 360 \textbf{ girls}$$

3) There are 60 less calves than cows in a farm and the quotient of cows to calves is 13. How many calves and cows are there?

4) Emma is 28 years older than his daughter. If their quotient is 8, how old is each?

5) Tom is 4 times as old as his grand child. If the difference between their ages is 60 years, how old is each?

6) Mugisha has 200 more goats than cows in a farm. The number of goats is seven times the number of cows. Find the number of goats and cows.

7) Mukankusi sells 72 more apples than oranges every day. If there is 4. Find the numbers.

8) The difference between two numbers is 56 and their quotient is 9. Find the numbers.

9) There are 20 more mangoes than oranges in a basket. The number of mangoes is 6 times the number of oranges. How many mangoes and oranges are there?

4. SHARES IN RATIOS

Definition: A ratio is the relation between two or more quantities by division.

Notice: * Ratios are written in their lowest terms.

      * Ratios must have no units.

a) **Expressing quantities as ratios**

The ratio of x to **y** is represented as $\frac{x}{y}$ where **y** is not zero. This can be written as $x \div y$ and is read as x to **y**.

Examples

1) In a family there are 4 girls and 5 boys.

   a) Express the number of girls as a ratio of boys.

   b) What is the ratio of boys to girls?

Solution

   a) Fraction $= \frac{4}{5}$

   The ratio of girls to boys is 4:5

   b) Fraction $= \frac{5}{4}$

   The ratio of boys to girls is 5:4

2) There were 200 people at a graduation party. 80 were females and the rest males.

   a) What is the ratio of females to males?

   b) What is the ratio of males to females?

Solution

   Total = 200

   Females = 80

   Males = 120

   a) Fraction $= \frac{80}{200} = \frac{2}{5}$

   The ratio is 2:5

   b) Fraction $= \frac{120}{200} = \frac{3}{5}$

   The ratio is 5:3

b) **Expressing ratios to their lowest terms**

1) Expressing 12:15 in its lowest terms.

   Fraction $= \frac{12}{15} = \frac{4}{5}$

   The ratio is 4:5

2) In a class of 50 pupils, there are 20 boys. What is the ratio of boys to girls?

   Fraction $= \frac{20}{30} = \frac{2}{3}$

   The ratio is 2:3

3) Expressing 20 minutes as a ratio of 1 hour.

   Fraction $= \frac{20\ min}{1\ hr} = \frac{20\ min}{60\ min} = \frac{1}{3}$

The ratio is 1:3

4) What is the ratio of 20 cm to 2m?

5) Expressing 0.4ha as a ratio of 60dam^2.

6) In a box of 120 pens, there are 45 red pens.

   a) What is the ratio of red pens to blue pens?

   b) What is the ratio of blue pens to red pens?

c) Increasing a quantity in a given ratio

1) Increase 80 kg of rice in the ratio of 5:4

   Fraction: $\frac{5}{4}$

   80kg $\times \frac{5}{4}$ **= 100 kg**

2) Increase 6000 in the ratio of 3:10

   Fraction: $\frac{10}{3}$

   6000 $\times \frac{10}{3}$ **= 20 000**

3) The price of a cow increased in the ratio of 3:2. If the price of a cow was 400,000 F, find the new price of a cow.

4) There 500 pupils at Murambo primary school. The number was increased to the ratio of 8:5. How many pupils are there now?

5) The salary of Helen increased in the ratio of 5:3. Find her new salary if the old salary is 90,000 Frw.

d) Decreasing a quantity in a given ratio

1) Decrease 1,500 F in the ratio of 2:3

   Fraction: $\frac{2}{3}$

   1500 $\times \frac{2}{3}$ **= 1000**

2) Decrease 480 kg of sugar in the ratio of 3:4

   Fraction: $\frac{3}{4}$

   480 kg $\times \frac{3}{4}$ **= 360 kg**

3) In a class of 90 pupils, the number decreased in the ratio of 3:5. Find the new number of pupils.

4) A man's salary decreased in the ratio of 1:2. What is the new salary if the old salary is 30,000 Frw?

e) Finding the ratio increase or ratio decrease

1) The number of pupils in a school increased from 600 to 800. In what ratio did the number increase?

$$\textbf{Fraction: } \frac{new\ number}{old\ number} = \frac{800}{600} = \frac{3}{4}$$

The ratio is 3:4

2) By what ratio must 800 be decreased to be 400?

$$\textbf{Fraction: } \frac{new\ number}{old\ number} = \frac{400}{800} = \frac{1}{2}$$

The ratio is 1:2

3) The weight of cassava to be mixed for millet bread was decreased from 15 kg to 5 kg. In what ratio did the weight decrease?

4) The price of a book is increased from 10 000 F to 12,000 F. In what ratio did the price increase?

5) The number of pupils in a class decreased from 40 to 30. In what ratio was it decrease?

6) There was a reduction in price of a radio from 48,000 Frw to 32,000 Frw. In what ratio was the price of the radio reduced?

f) **Finding the old number**

1) The price of an article increased in the ratio of 5:3. If the new price of the article is 5,000F, what was the old price of the article?

Fraction $= \dfrac{3}{5}$

$\dfrac{3}{5} \times$ **5,000 Frw = 3,000 Frw**

2) A number decreased in the ratio of 2:7 become 6000. What was the number?

Fraction $= \dfrac{7}{2}$

$\dfrac{7}{2} \times$ **6,000 = 21,000**

3) Which number when increase in 5:2 becomes 2,000?

4) The number of pupils at a certain school increased in the ratio of 10:7. If the school has now 3 000 pupils, how many pupils were there last year?

5) Find the number when decreased in the ratio of 11:15 becomes 220.

g) Sharing quantities by a given ratio

1) Share 10,000 F between Sano and Helen in the ratio of 2:3

Sum = 10,000 F

Sum of ratios = 2+3 = 5

1 ratio = 10,000 F ÷ 5 = 2,000 F

Sano = 2 x 2,000 F = 4,000 F

Helen = 3 x 2,000 F = 6,000 F

2) Kamanzi, Umwali and Muteteri contributed 48,000 Frw for a business in a ratio of 2:3:5 respectively. How much did each contribute?

Sum = 48,000 F

Sum of ratios = 2+3+5= 10

1 ratio = 48,000 F ÷ 10 = 4,800 F

Kamanzi = 2 x 4,800 F = 9,600 F

Umwali = 3 x 4,800 F = 14,400 F

Muteteri = 5 x 4,800 F = 24,000 F

3) Divide 72 000 F in 4:3:1 respectively.

4) Two schools A and B got a donation of 800 000 Frw. The money was shared in a ratio of 3:5 respectively. How much did each school get?

5) James and Felix shared 60 mangoes in 3 to 1 part respectively. How many mangoes did each get?

6) Share 360,000 Frw among three people in the ratio of 3:7:2 respectively.

7) Divide 80 kg of rice in 7 to 3 to 5 parts respectively.

h) Problems involving ratios

1) The ratio of boys to girls in a class is 3:4. If there are 24 girls, how many boys are there?

Girls = 24

Ratio of girls= 4

1 ratio = 24 ÷ 4 = 6

Boys = 3 x 6 = 18

2) There are 40 less cows than goats in a farm. If the ratio of cows to goats is 3 to 7 respectively, how many cows and goats are there?

Difference = 40

Difference of ratios = 7 – 3 = 4

1 ratio = 40 ÷ 4 = 10

Cows = 3 x 10 = 30 cows

Goats = 7 x 10 = 70 goats

i) More about sharing using ratios

1) Ben, Tom and Betty shared some money in the ratio of 4:3:5 respectively. If Tom received 12,000 Frw, how much money did others get?

Tom's share = 12,000 F

Tom's ratio = 3

1 ratio = 12,000 F ÷ 3 = 4,000 F

Ben = 4 x 4,000 F = 16,000 F

Betty = 5 x 4,000 F = 20,000 F

2) Amani, Louis and David shared some kg of beans in 2 to 5 to 3 parts respectively. If David got 10 kg less than Louis, how many kg did each get?

Difference between David and Louis = 10 kg

Difference of ratios = 5 – 3 = 2

1 ratio 10 kg ÷ 2 = 5 kg

Amani = 2 x 5 kg = 10 kg

Louis = 5 x 5 kg = 25 kg

David = 3 x 5 kg = 15 kg

3) Three people Agape, Peter and Celestin shared some money to start a business in 6:3:5 respectively. If Peter contributed 600,000 F,

a) Find the total amount of money that they all contributed.

b) How much money did Agape contribute?

c) How much money did Celestin contribute?

4) The angles of a rectangle are in the ratio of 3:5:1 respectively. Find the value of each angle.

5) Three families shared some kg of rice in 2 to 4 to 3 parts respectively. If the third person received 120 kg,

a) How many families did the rest families receive?

b) Find the total number of kg they shared.

6) Alloys, Nina and Manzi shared some money in the ratio of 3:4:5 respectively. If Manzi got 10,000 F,

a) How much money did others get?

b) Find the sum of money that they all shared.

7) Manzi, John and Theta shared some money in 2:5:3. If John got 9,000 Frw more than John,

a) Find the total amount of money they shared.

b) How much money did each get?

5. **When one is a fraction of another**

1) Share 3,600 frw between Kamanzi and Benitha such that kamanzi gets $\frac{2}{7}$ of Benitha's share.

   Sum = 3,600 Frw
   Sum of ratios = 2 + 7 = 9
   1 ratio = 3,600 Frw ÷ 9 = 400 Frw
   Kamanzi = 2 x 400 Frw = 800 Frw
   Benitha = 7 x 400 Frw = 2,800 Frw

2) David is 20 years younger than his sister and the sister is $\frac{3}{8}$ of David's age. How old is each?

   Difference = 20 years
   Difference of ratios = 8 – 3 = 5
   1 ratio = 20 ÷ 5 = 4
   Sister= 4 x 3 = 12 years
   David = 4 x 8 = 32 years

3) The difference between the father's age and his son is 36 years. If the son's age is $\frac{1}{4}$ of his father's age, fid their ages.

4) In two drums there are 385 litres of water. The capacity of the first drum is $\frac{5}{6}$ of the second. How many litres does each drum contain?

5) Divide 60,000 Frw between Valens and Wellars such that Valens gets $\frac{1}{4}$ of Wellars' share.

6) The son's age is $\frac{3}{8}$ the father's age. If the difference of their ages is 25 years, how old is each?

6. **Unequal shares involving more than two shares**

1) Agnes, Manzi and Thacianna shared 14,000 F. Agnes got 2,000 F more than manzi and Thacianna got twice as much as Manzi. How much money did each get?

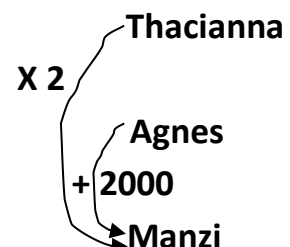   **Let x = Manzi's share**
   x+2000 = Agnes' share
   $2x$ = Thacianna's share
   4x+2000=14000
   4x = 14000 – 2000
   4x=12000
   $$\frac{4x}{4} = \frac{12000}{4}$$

x = 3,000

Manzi's share = x = 3,000 F

Agnes's share = x+2000 = 3,000 F + 2,000 F = 5,000 F

Mahoro's share = $2x$ = 2 x 3,000 F = 6,000 F

2) Share 32,000 F among Thomas, Kabano and Aloys such that Kabano gets 3,000 F less than Thomas and Aloys gets thrice as much as Thomas. How much can each get?

Let x = Kabano's share

x+3000 = Thomas' share

$3x$+9000 =Aloys' share

$5x$+12000=32000

$5x$ = 32000 − 12000

$5x$=20000

$$\frac{5x}{5} = \frac{20000}{5}$$

x = 4,000

Kabano's share = x = 4,000 F

Thomas' share = x+3000 = 4,000 F + 3,000 F = 7,000 F

Aloys' share = $3x$+9000 = (3 x 4,000)+9000= 12,000 F + 9000 F

= 21,000 F
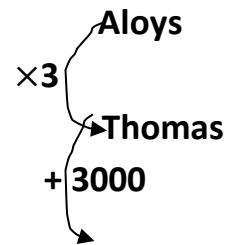
3) Divide 15,000 Frw between Arthur, Benedict and Charles such that Benedict gets 2,000 Frw more than Arthur and Charles gets 1,000Frw less than benedict.

4) Fulgence, Valentin and Oscar shared 45,000 Frw such that Valentin got twice as much as Fulgence and Oscar got thrice as much as Valentin. How much did each get?

5) Three men shared 80,000 Frw as follows: Maritha got double as much as Diana, John got 8,000 Frw more than Maritha. How much did each get?

7. <u>More about unequal shares</u>

1) Karimba's age is 3 times Rukundo's age. If the their total age is 20 years, find how many times Karimba will be as old as Rukundo in 5 years time.

*Now: Sum = 20 years

Q = 3

Rukundo = Sum ÷(Q + 1) =20 ÷(3 + 1) = 20 ÷ 4 = 5

Rukundo is 5 years

Karimba is 5 x 3 = 15 years

*In 5 years – Rukundo = 5 + 5 = 10 years

– Karimba = 15 + 5 = 20 years

Number of times = $\frac{20\ yaers}{10\ years}$ **= 2 times**

2) Manzi is 45 years older than Peter. If their total age is 85 years, how many times will Manzi be as old as Peter in 10 years?

3) Alloys is 25 years younger than his father and their total age is 85 years. How many times was the father as many as Aloys in 5 years ago?

TOPIC 7: <u>PROPORTION</u>

1) <u>Simple direct proportion</u>

This is the relation between two quantities whereby if one quantity **increases** another also **increases** and when one **decreases** the second also **decreases.**

<u>Examples</u>

1) 4 pens cost 2,000 frw. What is the cost of 7 similar pens?

4 pens $\longrightarrow$ $\frac{2000\ F \times 7}{4}$ **= 3,500 Frw**

7 pens

2) Given that P varies directly proportional as Q and P = 10 when Q = 6, find the value of P when Q = 48.

Q	P
6	$\frac{10 \times 48}{6}$ **= 80**
48	

3) A dozen of books cost 36,000 Frw. Find the cost of 26 books.

4) 2 trays of eggs cost 4,800 Frw. How many eggs can one buy with 3,200 Frw?

5) 6 men eat 3 kg of beans a day. How many kg can 18 men eat?

6) 50 kg of meat cost 10,000 frw. Find the cost of 30 kg of meat.

7) 20 science books cost 60,000 Frw. How many books can you buy with 45,000 Frw?

8) Given A varies directly as B and A=6 when B=8. Find the value of A when B=4.

9) There are 24 students in a class. Every child contributed 1,500 Frw for a trip.
   a) How much money did they contribute altogether?
   b) How many students paid 180,000 Frw?

1) <u>Simple direct proportion</u>

This is the relation between two quantities whereby if one quantity **increases** another **decreases** and when one **decreases** the second **increases.**

<u>Examples</u>

1) 6 men do some work in 12 days. How long will 9 men take to do that work?

6 men $\longrightarrow$ $\frac{12\ days \times 6}{9}$ **= 8 days**

9 men

2) Given that P varies inversely proportional as Q and P = 12 when Q=10. Find the value of Q when P=6.

$$\textbf{P} \qquad\qquad\qquad \textbf{Q}$$

12 ⟶ $\dfrac{10 \times 12}{6}$ **= 20**

6

3) 20 people have enough food for 5 days. How long will that food last if there are 10 people?

4) 6 children mow a school garden in 24 hours. How many children mow the same garden in 18 hours? (Assume all children work at the same rate)

5) 8 girls paint a house in 12 days. How many girls can paint the same house in 16 days?

6) 15 boys have enough food for 30 days. How many boys can eat the same food in 20 days?

7) A car travelling at 80km/h covers a certain distance in 5 hours. What will be its speed if that distance is to be covered in 4 hours?

8) If R varies inversely proportional as M and R=16 when M=15, find the value of R when M=30.

More about simple proportion (Part I)

1) A school of 300 students has enough food for 90 days. How long will that food last if 60 more students join the school?

300 students ⟶ $\dfrac{90 \; days \times 300}{360}$ **= 75 days**

300+60=360students

2) A cooperative of 10 people receive a donation of 400,000 Frw each year. How much money will they receive if 40 people left the cooperative?

3) A district of 40 schools is given 1200 books each year. How many books will the district be given if 20 more schools open this year?

4) A school of 600 pupils had enough food for 80 days. How long will the food last if 100 pupils were chases from the school?

More about simple proportion (Part II)

1) 1 500 people have food enough to last for 40 days. After 10 days their number increased by 300 people. How many days will the food last if the individual share will not change?

1500 people $\longrightarrow$ $\dfrac{30\ days \times 1500}{1800}$ **= 25 days**

1500+300=1800 people

2) A school of 600 pupils had enough food for 50 days. After 20 days, 200 pupils left the school and joined another school. How long did the food last?

3) 900 people have food enough to last for 80 days. After 30 days, 600 more people joined the school. For how long will the food last?

4) 100 men can do some work in 90 days. After working for 30 days, 50 more men joined them. How long did they take to finish the work?

4) Compound proportion

Compound proportion deals with more than two variables.

Examples

1) 20 boys who work 6 hours a day can do 100 m in 20 days. How many boys will be needed to complete 50 m in 4 days if they work 12 hours a day?

20 days $\longrightarrow$ **100 m** $\longrightarrow$ **6 h** $\longrightarrow$ $\dfrac{20\ boys \times 20 \times 50 \times 6}{4 \times 100 \times 12}$ **= 64m**

4 days $\longrightarrow$ **50 m** $\longrightarrow$ **12h**

2) 30 workers working 4 hours a day can make 80 shirts in 54 days. How long would they take if there are 18 workers working 9 hours a day to make 120 shirts?

20 workers $\longrightarrow$ **4 hrs** $\longrightarrow$ **80 shits** $\longrightarrow$ $\dfrac{54\ days \times 30 \times 4 \times 120}{18 \times 9 \times 80}$ **= 60 days**

18 workers $\longrightarrow$ **9 hrs** $\longrightarrow$ **120 shirts**

3) If 15 men working 9 hours a day dig 60 m, how many metres will 18 men dig working 8 hours a day?

4) 15 tailors working 30 days make 150 dresses. How many dresses will 12 tailors make if they work for 40 days?

5) 6 men working 7 hours a day can do some work in 10 days. How long will 5 men working 4 hours a day take to do the same work?

TOPIC 8: BUYING AND SELLING

1) Cost price or buying price or purchase price

1) Kabera went to the market and bought 60 kg of rice at 800 Frw per kg. Find the cost price.

Cost price = 60 x 800 F = 40,000 F

2) John wants to buy 200 kg of rice at 1,000 frw per kg. How much will he pay?

3) In the market two traders are selling tomatoes. The first trader sells a 3 kg bucket for 273 Frw. The second trader sells a 5 kg bucket for 500 Frw. Which is offering better value?

4) How much money can a person pay 8 litres of oil if the cost of 1 litre is 2500 frw.

2) Selling price

1) John bought 50 kg of irish potatoes and sold them at 300 frw per kg. How much will he pay?

50 x 300 F = 15,000 Frw

2) Alloys sold his 4 cows at 300,000 frw, 250,000 frw, 350,000 frw and 200,000 frw. Find his selling price.

3) Kanakuze bought 6 crates of soda and sold them at 7400 Frw each. Find the selling price.

3) Profit or benefit

1) A dozen of books bought at 2,000 Frw each was sold at 2,500 Frw each. Find the profit.

CP = 2000F x 12 = 24,000 Frw
SP = 2500F x 12 = 30,000 Frw
Profit = SP – CP = 30,000 Frw – 24,000 Frw = 6,000 Frw

2) Manzi bought 40kg of rice at 800frw per kg and sold them at 900frw per kg. Find his profit.

3) A goat sold at 36,000frw had been bought at 33,000frw. What was the benefit?

4) Loss

1) Samson bought a radio at 85,000Frw and sold them at 80,000Frw. Find his loss.

CP = 85,000 Frw
SP = 80,000 Frw
Loss = CP – SP = 85,000 – 80,000 = 5,000FRW

2) Amanda bought 50kg of sugar at 750Frw per kg and sold them at 720Frw. What was the loss?

3) Kabera bought TV set for 215,000Frw and sold it for 197,000Frw. Find the loss made.

5) Finding the cost price

1) Martin sold a cow for 400,000Frw and made a profit of 50,000Frw. What was the cost price?

SP = 400,000 Frw

Profit = 50,000Frw

CP=SP − Profit = 400,000 Frw − 50,000Frw = 350,000 Frw

2) Ben sold his land for 2,600,000Frw and made a loss of 350,000Frw. Find the cost price.

SP = 2,600,000 Frw

Loss = 350,000Frw

CP=SP + Profit = 2,600,000 Frw + 350,000Frw = 2,950,000 Frw

3) James sold 30kg of beans at 400Frw per kilogram and sold them at by making a profit of 1,200 Frw. Find the cost price.

4) Ana sold 60kg of wheat at 300Frw per kg and made a loss of 3,000Frw. How much did he buy one kilogram of wheat?

5) David had 20l of juice and sold them at 1,000Frw per litre. If he made a profit of 2,000Frw, what was the cost price of 1 kg?

6) Finding selling price

1) Karenzo bought a goat at 40,000 Frw. He made a profit of 3,000 Frw. What was the selling price?   **CP = 40,000 Frw**

Profit = 3,000Frw

SP = CP + Profit = 40,000 Frw + 3,000Frw = 43,000 Frw

2) Doris bought 80kg of rice at 600Frw and sold them by making a loss of 4,000Frw. For how much did he sell a kilo of rice?

CP = 80 x 600 = 48,000Frw

Loss = 4,000Frw

SP = CP − Loss = 48,000 − 4,000 = 44,000 Frw

1 kg = $\frac{44,000\ frw}{80}$ = 550 F/kg

3) Ndori bought meat for 32,000Frw and sold them by making a loss of 10,000Frw. Find the selling price.

4) Sandra bought 70 kg of maize at 200F/kg. He sold them by making a profit of 2,000Frw. How much did he sell one kg of maize?

7) Percentage profit

1) A trader bought a hen for 8,000 frw and sold it at 10,000 frw. Find his percentage profit.   **CP = 8,000 Frw**

SP = 10,000 Frw

Profit = 10,000 F − 8,000 F = 2,000 Frw

Fraction = $\frac{Profit}{CP} = \frac{2,000\ Frw}{8,000\ Frw} = \frac{1}{4}$

$$\% \text{ profit} = \frac{1}{4} \times 100\% = 25\%$$

2) Kamanzi bought 40kg of beans at 400Frw and sold them at 500Frw. What was the percentage profit?

8) Percentage loss

1) A television set that was bought at 80,000F, has been sold at 70,000F. Calculate the percentage loss.

> **CP = 8,000 Frw**
>
> **SP = 10,000 Frw**
>
> **Profit = 10,000 F – 8,000 F = 2,000 Frw**
>
> **Fraction** $= \dfrac{Profit}{CP} = \dfrac{2,000\ Frw}{8,000\ Frw} = \dfrac{1}{4}$
>
> $\% \text{ profit} = \dfrac{1}{4} \times 100\% = 25\%$

2) Mugeni bought a sheep at 24,000Frw and sold it at 20,000Frw. Find the percentage loss.

TOPIC 8: PERCENTAGES

1) The meaning of percentages

❖ Percent means **"per hundred"** or **"out of one hundred"** or **"for every hundred"**.

❖ A percentage is a fraction whose denominator is **100.**

❖ If we say 60 percent of the class are girls, we mean 60 out of 100 in the class are girls.

❖ This can be represented as **60 out of 100** or $\dfrac{60}{100}$ or **60%**

❖ The symbol of percentage is **%**

2) Expressing decimals as percentages

Express the following decimals as percentages:

a) $0.35 = \dfrac{35}{100} \times 100\%$

   $= 35\%$

b) $0.6 = \dfrac{6}{10} \times 100\%$

   $= 60\%$

c) $3.85 = \dfrac{385}{100} \times 100\%$

   $= 385\%$

d) $0.236 = \dfrac{236}{1000} \times 100\%$

   $= 23.6\%$

e) $3.5 = \dfrac{35}{10} \times 100\%$

= 350%

3) Changing percentages to decimals

Express the following percentages as decimals:

a) $45\% = \dfrac{45}{100}$

  = 0.45

b) $29\% = \dfrac{29}{100}$

  = 0.29

c) $146\% = \dfrac{146}{100}$

  = 1.46

d) $7\% = \dfrac{7}{100}$

  = 0.07

e) $6.8\% = \dfrac{6.8}{100}$

  = 0.068

4) Expressing fractions as percentages

Write the fractions below as percentages:

a) $\dfrac{18}{100} = \dfrac{18}{100} \times 100\%$

  = 18%

b) $\dfrac{1}{2} = \dfrac{1}{2} \times 100\%$

  = 50%

c) $\dfrac{1}{4} = \dfrac{1}{4} \times 100\%$

  = 25%

d) $\dfrac{7}{10} = \dfrac{7}{10} \times 100\%$

  = 70%

e) $2\dfrac{1}{4} = \dfrac{9}{4} \times 100\%$

  =225%

f) $\dfrac{7}{10} = \dfrac{7}{10} \times 100\%$

  = 70%

g) $1\dfrac{2}{5} = \dfrac{7}{5} \times 100\%$

  = 140%

5) Expressing percentages as fractions

Express the following percentages as fractions in their lowest terms:

a) $65\% = \dfrac{65}{100}$

$= \dfrac{13}{20}$

b) $40\% = \dfrac{40}{100}$

$= \dfrac{2}{5}$

c) $75\% = \dfrac{75}{100}$

$= \dfrac{3}{4}$

d) $2\dfrac{1}{4}\% = \dfrac{9}{4}\% = \dfrac{\frac{9}{4}}{100}\%$

$= \dfrac{9}{4} \div 100$

$= \dfrac{9}{4} \times \dfrac{1}{100} = \dfrac{9}{400}$

e) $150\% = \dfrac{150}{100} = \dfrac{3}{2}$

$= 1\dfrac{1}{2}$

6) Expressing ratios as percentages

a) Express 3:5 as a percentage.

Fraction $= \dfrac{3}{5}$

$\% = \dfrac{3}{5} \times 100\% = 60\%$

b) Express 3:2 as a percentage

Fraction $= \dfrac{3}{2}$

$\% = \dfrac{3}{2} \times 100\% = 150\%$

7) Expressing percentages as ratios

Change the following percentages into ratios:

a) $35\% = \dfrac{35}{100}$

$= \dfrac{7}{20}$

The ratio is 7:20

b) $50\% = \dfrac{50}{100}$

$= \dfrac{1}{2}$

The ratio is 1:2

c) $250\% = \dfrac{250}{100}$

$$= \frac{5}{2}$$

The ratio is 5:2

8) Expressing quantities as percentages

1) Anita scored 30 marks out 40 in a test. Express her marks as a percentage.

Fraction $= \frac{30}{40} = \frac{3}{4}$

$\% = \frac{3}{4}$ **x 100% = 75%**

2) There are 50 males and 30 females in a cooperative. Find the percentage of males.

Total = 50 + 30 = 80

Fraction of males $= \frac{50}{80} = \frac{5}{8}$

$\% = \frac{5}{8}$ **x 100% = 62.5%**

3) In a class of 50 pupils, 20 are girls.

a) What percentage of the class are girls?

b) What percentage of the class are boys?

4) There were 800 desks in a school. 200 desks are broken. Find the percentage of the remaining desks.

9) Finding parts of a percentage

1) If 60% of a class are girls, what percentage are boys?

Total = 100%

Girls = 60%

Boys = 100% - 60% = 40%

2) Joseph did 30% of his weekend homework on Saturday and 55% on Sunday. What percentage represents the undone questions?

3) In a school 87% of the pupils are present. What percentage are absent?

4) Mutoni sold 27% of the cows she had. Find the percentage of the remaining cows.

10) Finding quantities equivalent to a percentage

1) 5% of the learners in P6 are boys. If the class has 80 leaners, how many boys are there?

100%

5%

$\frac{80\ learners \times 5}{100}$ **= 4 boys**

2) 20% of a number is 700. What is the number?

20%

100%

$\frac{700 \times 100}{20}$ **= 3,500**

3) In a box of pieces of chalk, 40% are white. If there are 60 white pieces of chalk, find the total number of pieces of chalk that are in the box.

40% $\longrightarrow$ $\dfrac{60 \text{ pieces} \times 100}{40}$ **= 150 pieces**

100%

4) What is 30% of 6,000Frw?

30% of 6,000Frw = $\dfrac{30}{100} \times 6,000Frw$ **= 1,800 Frw**

5) 45% of the fish in a pond are catfish. There 900 catfish. How many fish are in the pond altogether?

6) In a class of 40 pupils, 60% are boys. How many boys and girls are there?

7) Find 25% of 500 m^2.

8) $37\frac{1}{3}$% of the school are females. If there are 224 females, how many males are there?

9) 60 is equivalent to 10% of a number. Find the number.

10) In Cyanika primary school, there are 800 pupils. If 40% are girls,
   a) How many girls are there?
   b) How many boys are there?

11) Sharing quantities using percentages

1) Share 40,000 Frw between Peter and Felix such that Peter gets 30%.

Peter = 30% of 40,000Frw

= $\dfrac{30}{100} \times$ **40,000 Frw = 12,000 Frw**

Felix = 70% of 40,000Frw

= $\dfrac{70}{100} \times$ **40,000 Frw = 28,000 Frw**

2) Share 80 kg of rice among Sano, Helen and James such that Helen gets 20% and Sano gets 40%.

3) Three families shared 200,000 frw such that the first got 40% and the second got 10%.

12) Increasing quantities by percentages

1) Increase 2,000 frw by 20%

Old number = 2,000 F

% increase = 20%

Old number = 100%

New number = 100% + 20% = 120%

100% ⟶ $\dfrac{2{,}000\ F \times 120}{100}$ = **2,400 f**

120%

2) The monthly salary of Kanakuze was 250,000 frw. If it was increased by 10%, find her current monthly salary.

3) At Murambo primary school there were 1,200 pupils last year. This year the number has been increased by 15%. How many pupils are there now?

4) Increase 50,000 by 12%

13) <u>Decreasing quantities by percentages</u>

1) Decrease 4,000 by 30%

        Old number = 4,000

        % decrease = 30%

        Old number = 100%

        New number = 100% - 30% = 70%

        100% ⟶ $\dfrac{4{,}000\ F \times 70}{100}$ = **2,800 f**

        70%

2) Reduce 250 kg of rice by 10%

3) Duterimbere cooperative had 500 members. If 10% left the cooperative, how many members does the cooperative have now?

14) <u>Finding the percentage increase or percentage decrease</u>

1) The number of students at Susa primary school increased from 700 to 840. By what percentage did it increase?

        Increment = 840 – 700 = 140

        Fraction = $\dfrac{increment}{old\ number} = \dfrac{140}{700} = \dfrac{1}{5}$

        % = $\dfrac{1}{5}$ **× 100% = 20%**

2) By what percentage can 8,000 kg be decreased to become 6,400 kg?

3) When 5,000 is increased by x% it becomes 5,500. Find the value of x.

4) There were 400 pupils at a school last year. There are 480 pupils this year. What is the percentage increase of the number of pupils?

15) <u>Finding the original number after percentage increase or decrease</u>

1) A number increased by 10% become 4,400. What is the number?

        New number = 4,400

        % increase = 10%

        Old number = 100%

New number = 100% + 20% = 110%

110% $\longrightarrow \dfrac{4,400 \times 100}{110}$ **= 4,000**

100%

2) A trader sold a goat at 36,000 Frw and made a loss of 20%. What was the cost price of the goat?

        SP = 36,000 Frw

        % loss = 20%

        CP = 100%

        SP = 100% - 20% = 80%

        80% $\longrightarrow \dfrac{36,000 \times 100}{80}$ **= 45,000 Frw**

        100%

3) Jane's salary was increased by 7%. His new salary is 64,200 F. what was her salary?

4) John sold his radio for 11,000 Frw and made a profit of 10%. What was the cost price?

5) After increasing a number by 5% it became 285. What was the original number?

6) What number when decreased by 9% becomes 36,400?

7) Akim's salary was increased to 540,000 Frw by 8%. Find his old salary.

8) Justine's monthly salary was increased by $7\frac{1}{3}$% to 86,000Frw. Calculate his previous salary.

9) By selling a pair of shoes at 90,000 Frw a dealer made a loss of 10%. Calculate the cost price.

16) **More about percentages**

1) Express 40 as a percentage of 80.

        Fraction $= \dfrac{40}{80} = \dfrac{1}{2}$

        % $= \dfrac{1}{2} \times$ 100% = 50%

2) Express $\frac{2}{5}$ of 20 as a ratio of 32.

        Fraction $= \dfrac{\frac{2}{5}\,of\,20}{32} = \dfrac{\frac{2}{5}\times 20}{32} = \dfrac{8}{32} = \dfrac{1}{4}$

        % $= \dfrac{1}{4} \times$ 100% = 25%

3) If 20% of a number is 200. What is the number?

        20% $\longrightarrow \dfrac{200 \times 100}{20}$ **= 1,000**

        100%

4) If 40% of a number is 2,000, what is 60% of the same number?

        40% $\longrightarrow \dfrac{2,000 \times 60}{40}$ **= 3,000**

60%

5) Express 2 km as a percentage of 400m.

6) Express 30 minutes as a percentage of 2 hours.

7) Express 200 cm³ as a percentage of 4 litres.

8) If 6% of a number is 15,600. Find the number.

9) If 20% of a number is 120. Find the number.

10) Express 45 minutes as a percentage of 1 hour.

17) Applications of percentages

A) Discount

A discount is a deduction or a reduction made from the market price, allowed to a customer.

A discount is obtained when an article is sold at a price below the market price.

Examples

1) The market price of a dress was 9,000 Frw. Jane bargained for it and the shopkeeper accepted 8,750 Frw. Calculate the discount the shopkeeper allowed Jane.

   Market price = 9,000 Frw

   Selling price = 8,750 Frw

   Discount = MP – SP = 9,000 Frw – 8,750 Frw = 250 Frw

2) After being allowed a discount of 3,800 frw, Munyana paid 49,600 Frw for a radio. What was the market price of the radio?

3) The market price of a car was 4,000,000 Frw. Musoni was allowed a discount of 500,000 Frw. What was the selling price?

B) Percentage discount

1) The market price of a shirt was 7,000 Frw. Moses paid 6,860 Frw after being allowed a discount. Calculate the percentage discount.

   MP = 7,000 Frw

   SP = 6,860 Frw

   Discount = MP – SP = 7,000 Frw – 6,860 Frw = 140 F

   Fraction $= \dfrac{discount}{MP} = \dfrac{140}{7000} = \dfrac{1}{50}$

   % discount $= \dfrac{1}{50} \times 100\% = 2\%$

2) A customer paid 60,000 Frw for a television set. The market price of the television set was 64,000 Frw. Calculate the percentage discount.

3) The market price of a bed was 20,000 Frw. Sarah was given a discount of 3%.
   a) How much was the discount?
   b) How much did the customer pay?

C) Commission

A commission is payment made to a sales representative or a sales agent.
The payment is calculated as a percentage of the sales.

Examples

1) Muvunnyi was a sales agent. He sold goods worth 145,000 Frw and was paid a commission of 5% of the sales. How much money did he get?

Commission = 5% of 145,000 Frw

$$= \frac{5}{100} \times 145,000 \text{ frw}$$

= 7,250 Frw

2) A newspaper vendor was paid a commission of 2% on the sales he made. He sold 280 copies, each at 1,500 Frw. How much commission did he earn?

3) Promise is paid a salary of 80,000 Frw plus a commission of $3\frac{1}{2}$ on sales. In a month she sold goods worth 120,000 Frw. How much did she earn altogether?

4) A sales agent is paid a commission of 8% on his sales above 600,000 frw. He sold goods worth 1,100,000 Frw. How much commission did the sales agent earn?

D) Hire purchase

Hire purchase is a system of buying where someone first pays only a fraction of the total amount and then keeps on paying regularly installments until the whole amount is paid.

The first payment is called the **deposit.**

> ## Hire purchase = Deposit + Total installments

Examples

1) The market price of a TV is 100,000 Frw. It is also possible to buy that TV on hire purchase terms by paying a deposit of 40,000 Frw plus a monthly installment of 10,000 Frw for 9 months.
   a) Calculate the hire purchase cost.
   b) Promise buys the TV on hire purchase terms. How much extra does she pay?

Deposit = 40,000 F
Total installments = 10,000 F x 9 = 90,000 F
a) Hire purchase = Deposit + Total installments
= 40,000 Frw + 90,000 Frw
= 130,000 Frw
b) Extra cost = Hire purchase – Market price
= 130,000 Frw – 100,000 Frw
= 30,000 Frw

2) A car costs 6,000,000 Frw if you pay cash. It is also possible (available) on hire purchase terms by paying a deposit of 3,200,000 Frw and then 200,000 Frw each month for 2 years.
   a) Find the hire purchase price.
   b) Nice buys a car on hire purchase price. How much extra does she pay?

c) Express the extra cost as a percentage of the market price.

10) <u>TAXES</u>

1) Mark bought 200 kg of rice at 800 Frw and sold them at 1,000 Frw per kg. If he paid 10% of taxes on the selling price. Find his profit.

Cost price = 200 kg x 800 F/kg = 160,000 F

Selling price = 200 kg x 1,000 F/kg = 20,000 F

Taxes = 10% of 200,000 F = $\frac{10}{100}$ x 200,000 Frw = 20,000 F

Profit = SP – (CP + Profit)

      = 200,000 F – (160,000 F + 20,000 F)

      = 200,000 F – 180,000 F

      = 20,000 F

2) David sells cars worth 90,000 Frw on behalf of the importer. He receives a commission of 10% on the first 50,000,000 Frw and 3% on the remainder. How much does the importer receive if he pays 5% tax?

3) The cost of a new bicycle is 40,000 Frw. In order to buy it 8% of the cost must be paid to RRA. If the buyer had 50,000Frw, what balance would he remain with?

TOPIC 9: <u>POWERS AND INDICES</u>

1) <u>Definition of base and indices (exponents)</u>

The result of multiplying two or more numbers is a product. When the same number is used as a factor several times, we use **exponents (indices)** to simplify the writing of a product.

$$2 \times 2 \times 2 \times 2 = 2^4$$

Exponent → (top arrow of 2^4)

Power ← 2^4

Base → (base of 2^4)

Common factor (base)

❖ The common factor is called a **base. Exponent** (index) tells us the number of times a base is used as a factor.

❖ A number which is expressed using exponent is called a **power**.

2) <u>Writing powers or indices in words</u>

Write the following numbers in words:

- 5^2 = **five squared**

- 4^3 = **four cubed**
- 7^4 = **seven to the four power**
- 8^7 = **eight to the seventh power**
- 6^1 = **the first power of six**

3) <u>Writing products as powers and finding their value</u>

1) Write **3 x 3 x 3 x 3** in exponential form.

   3 x 3 x 3 x 3 = 3⁴

2) Write **4³** as a product of the same factor.

   4³ = 4 x 4 x 4

4) <u>Evaluating sums and differences of powers</u>

Evaluate the following:

a) $2^5 + 3^4$

   2⁵ + 3⁴ = (2 x 2 x 2 x 2 x 2) + (3 x 3 x 3 x 3)

   = 32 + 81 = 113

b) $(4^3) - (5^2)$

   (4³) – (5²) = (4 x 4 x 4) – (5 x 5)

   = 64 – 25 = 39

<u>Exercises</u>

c) $3^2 + 2^2 + 5^1$

d) $2^6 - 7^2$

e) $(4^3 - 2^4) - 2^5$

f) $(11^2 - 8^2) + 4^2$

g) $9^3 - (6^4 + 3^3)$

h) What is the sum of 2^3 and 2^5?

i) What is the difference between 7^3 and 8^2?

j) Given that y=3^2 and x=2^4. Find the value of:

   (a) x+y   **b)** y – x

5) <u>Multiplying powers of the same bases</u>

When multiplying two or more powers with the same bases, keep one base and add all exponents.

   Example: - $x^2 \times x^3 = x^{2+3}$

   $$= x^5$$

   - $y \times y^3 = y^{1+3}$

   $$= y^4$$

   - $2^3 \times 2^4 = 2^{3+4}$

   $$= 2^7$$

<u>Exercises</u>

1) Write the following expressions as single product.

   a) $2^3 \times 2^2$

   b) $3^3 \times 3^1$

   c) $7^1 \times 7^3 \times 7^2$

d) $q^2 \times q^4 \times q$

2) Evaluate the following:

   a) $2^3 \times 2^2 =$

   b) $10^2 \times 10^3 \times 10 =$

   c) $4^2 \times 4 \times 4^3 =$

3) Two powers have the same base. If one of them is 4^2 and the sum of their exponents in their product is 5,

   a) What is the second number?

   b) Write the exponential of their products.

6) **Dividing powers of the same bases**

When dividing two or more powers with the same base, keep one base and subtract the exponents.

Example: - $\frac{x^4}{x^2} = x^{4-2} = x^2$

- $y^4 \div y^3 = y^{4-3} = y^1$ or y

- $3^9 \div 3^6 = 3^{9-6}$ or $3^9 \div 3^5 = \frac{3^9}{3^5} = \frac{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3 \times 3 \times 3 \times 3} = 27$

  $= 3^3 = 27$

- $10^5 \div 10^3 = \frac{10^5}{10^3} = \frac{10 \times 10 \times 10 \times 10 \times 10}{10 \times 10 \times 10} = 100$

Exercises

Evaluate:

   a) $6^7 \div 6^5 =$

   b) $5^5 \div 5^2 =$

   c) $10^8 \div 10^7 =$

   d) $12^5 \div 12^3 =$

   e) $4^9 \div 4^7 =$

7) **Finding unknown using the law of multiplication and division of indices**

Solve:

1) $4^y \times 4^3 = 4^9$

   $4^y \times 4^3 = 4^9$

   $4^{y+3} = 4^9$

   y+3 = 9

   y = 9 − 3

   y = 6

2) $3^{2p} \times 3^2 = 3^4$

   $3^{2p} \times 3^2 = 3^4$

   $3^{2p+2} = 3^4$

   2p+2 = 4

   2p = 4 − 2

$\frac{2p}{2} = \frac{2}{2}$

P = 1

3) $5^{3m} \div 5^4 = 5^5$

   $5^{3m} \div 5^4 = 5^5$

   $5^{3m-4} = 5^5$

   3m − 4 = 5

   3m = 5 + 4

   $\frac{3m}{3} = \frac{9}{3}$

   m = 3

4) $8^y \times 4 = 32$

    $8^y \times 4 = 32$

    $2^{3y} \times 2^2 = 2^5$

    $2^{3y+2} = 2^5$

    $3y + 2 = 5$

$3y = 5 - 2$

$3y = 3$

$\dfrac{3y}{3} = \dfrac{3}{3}$

$Y = 1$

Exercises

Solve:

a) $7^{2y} \div 7^3 = 7^5$

b) $2^2 \times 2^m = 2^4$

c) $5^{3y} \div 5 = 5^3$

d) $3^7 \times 3^b = 3^{11}$

e) $11^4 \div 11^p = 11^5$

f) $x^a \times x = x^{11}$

g) $y^x \div y = y$

h) $8 \times 2^y = 16$

TOPIC 10: MIXTURES

A mixture is a combination of different things.

1) Finding the average price of 1 kg of the mixture

1) Manzi mixed 40 kg of beans of one type which cost 400 Frw per kg with 60kg of second type which cost 500 Frw per kg. Find the cost of 1 kg of the mixture.

    1^{st}type = $40 \times 400 = 16,000$

    2^{nd}type = $60 \times 500 = 30,000$

    Mixture = $100 \times x = 46,000$

        $100\,x = 46,000$

        $\dfrac{100\,x}{100} = \dfrac{46,000}{100}$

        $x = 460$ F/kg

    The cost of the mixture is 460F/kg

2) Andrew mixed 20kg of sorghum flour which cost 1,000F/kg with 30kg of millet flour which cost 2,000F/kg. Calculate the average price of the mixture.

3) Ana mixed 20 litres of local beer which cost 300 Frw per litre with 30litres of beer which cost 500 Frw per litre. Find the price of 1 litre of the mixture.

4) A man mixed 30kg of rice which cost 300 Frw per kg with 20kg of another type which cost 250 Frw per kg. How much did he sell each kg of mixed rice?

5) Kambanda has 12 kg of brown sugar which cost 1,100 Frw per kilogram and 8 kg of white sugar which cost 1,200 Frw per kg. What will be the average price of the mixture? Show your working out.

2) Finding the price of one type of the mixture

1) Sarah mixed 300 kg of beans which cost 1,000 Frw per kg with 400 kg of a different quantity. Find the price of the second type if she sold the mixture at 600 Frw per kg.

1st**type = 300 × 1,000 = 300,000**

2nd**type = 400 × x = 400x**

Mixture = 700 × 600 = 300,000 + 400x

420,000 = 300,000 + 400x

−400x = 300,000 − 420,000

−400x = −120,000

$$\frac{-400x}{400} = \frac{-120,000}{400}$$

x = 300 F/kg

The cost of the second is 300F/kg

2) Charles has 500 kg of mixed peas and sells them at 900 Frw per kg. If he has mixed 200 kg of one type that cost 1500 frw per kg, find the cost of 1 kg of the second type.

3) Kalisa mixed 20 kg of peas which cost 1,500 Frw per kg with 30 kg of a different type. Find the unit price of the second type if the mixture costs 900 Frw per kg.

4) Musoni has 30kg of mixed beans and he sells each kg at 700 Frw. If there are 10 kg of the type which cost 900 Frw per kg, find the price of one kg of the second type.

5) The information shows a mixture of two types of beans. Find the value of Y.

Types	Quantity	Price per kg
A	30 kg	1,400 Frw
B	50 kg	Y Frw

6) Ana mixed 120 kg of maize which cost 200 Frw per kg with 80 kg of another type and sold the mixture at 220 Frw per kg. Find the price of 1 kg of maize.

7) Musoni mixed 70 kg of millet flour which cost 240 Frw per kg with 80 kg of another type. Find the pice of 1 kg of the second type if the price of the mixture is 200 Frw per kg.

3) Finding the quantity of one type of mixture

1) Ngoga mixed two types of beans and sold a kg at 280 Frw. He had 4 kg of the first type which cost 400 Frw per kg and the second type which cost 200 Frw per kg. Find the quantity of the second type.

400   **80 (1^{st}type)**

280

200   **120 (2^{nd}type)**

80 ——————→ $\dfrac{4\ kg \times 120}{80}$ = **6 kg**

120
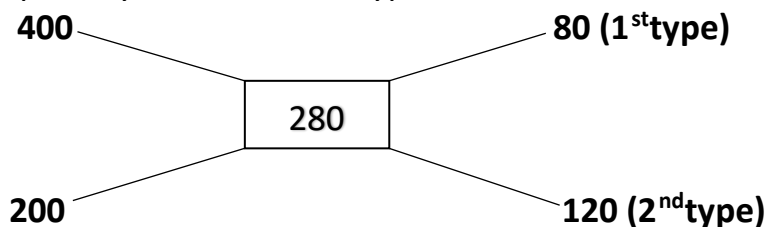
2) Aline mixed 50 kg of sugar which cost 500 Frw per kg and another type which cost 600 Frw per kg. If she sold the mixture at 560 Frw, find the quantity of the second type.

3) 24 kg of beans are mixed with maize. The cost of 1 kg of beans is 400 Frw and the cost of 1 kg of maize is 500 Frw, if the mixture costs 452 F per kg, find the number of kg of maize.

4) 10 kg of maize are mixed with sorghum. The cost of 1 kg of sorghum is 900 Frw, 1 kg of sorghum is 600 Frw and the cost of the mixture is 700 Frw per kg. Find the number of kg of sorghum.

5) The information shows two types of groundnuts and their costs per kg.

Types	Quantity	Price per kg
A	48 kg	800 Frw
B	X kg	1000 Frw

Find the value of X if the cost of 1 kg of mixture is 880 Frw.

4) **Finding the quantity of both types of mixture**

1) A trader mixed two types of beans and formed 30kg of the mixture which he sold at 700 Frw per kg. One type costs 900 Frw per kg whereas the second costs 600 Frw per kg. Find the quantity of each type.

900 ⟍ ⟋ **100 (1ˢᵗtype)**

| **700** |

600 ⟋ ⟍ **200 (2ⁿᵈtype)**

300 ——————→ $\dfrac{30\ kg \times 100}{300}$ = **10 kg**

100

1ˢᵗtype = 10 kg

2ⁿᵈtype = 30kg – 10 kg = 20kg
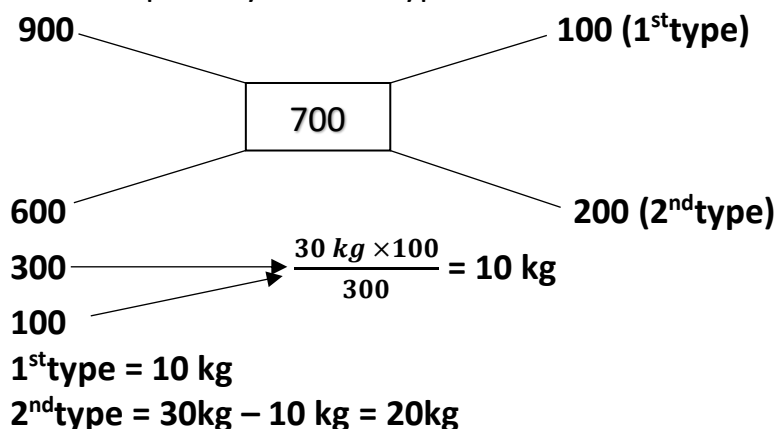
2) Mugabo had 150 kg of mixed beans that cost 260 Frw per kg. He had mixed two types of beans. One costs 290 Frw per kg and the second costs 240 Frw per kg. What was the weight of each type of bean in the mixture?

3) A shopkeeper mixed two types of sugar and formed 50 kg which he sold at 1,200 Frw per kg. If one type is sold at 1,500 Frw per kg and the second type at 1,000 Frw per kg. Find the number of kg of each type.

4) The information shows two types of beans and their cost per kg.

Types	Quantity	Price per kg
A	X kg	300 Frw
B	Y kg	360 Frw

What are the values of X and Y which should form 40 kg of mixture and cost 334.5 F per kilogram?

5) Ngarambe has 120 kg of a mixture of rice which he sells at 1000 Frw per kg. The 1st type is sold at 1,200 Frw per kg and the second type at 800 Frw per kg. Find the number of kg of each type.

TOPIC 11: <u>INTERESTS</u>
1. <u>Simple interest</u>

Definitions: * Principal or Capital (P): The money banked, borrowed or lent.

   * **Rate (R):** The percentage used to calculate interests.

   * **Time (T):** The period in years that the principal is invested.

   * **Interest (I):** The addition amount offered or paid back.

   * **Amount (A):** Total amount adding interests.

<u>Examples</u>

1) A farmer deposited 120,000 Frw in a bank that offers an interest rate of 10% per year. How much interest will the farmer get in 2 years?

   P = 120,000 Frw

   R = 10%

   T = 2 years

   $$I = \frac{P \times R \times T}{100 \times 1\ year} = \frac{120,000\ Frw \times 10 \times 2\ years}{100 \times 1\ year} = \textbf{24,000 Frw}$$

2) Sandrine deposited 120,000 Frw in a bank that gives $8\frac{1}{5}$% interest rate. Find the interest at the start of the fourth year.

3) Ines deposited 1,200,000 Frw in a bank at the rate of 10% per annum. Calculate Ines' interest after $2\frac{1}{2}$ years.

4) Calculate the simple interest on a loan of 80,000 Frw after 4 years at 8% interest rate per year.

5) Find the interest on 30,000 Frw at $2\frac{1}{2}$% after $5\frac{1}{4}$ years.

6) David borrowed 240,000 Frw from the bank that charges 12% interest rate per annum. How much interest did he pay at the start of the third year?

2. <u>More about simple interest</u>

1) Abayo banked 400,000 Frw in a bank that gives 6% interest rate p.a. Find the interest after 8 months.

P = 400,000 Frw

R = 6%

T = 8 months

$$I = \frac{P \times R \times T}{100 \times 1 \; year} = \frac{400,000 \; Frw \times 6 \times 8 \; months}{100 \times 12 \; months} = \textbf{16,000 Frw}$$

2) Find the simple interest on 72,000 Frw at $3\frac{1}{2}$% interest rate after 90 days.

P = 72,000 Frw

R = $12\frac{1}{2}$% = $\frac{25}{2}$%

T = 90 days

$$I = \frac{P \times R \times T}{100 \times 1 \; year} = \frac{72,000 \; Frw \times \frac{25}{2} \times 90 \; days}{100 \times 360 \; days} = \textbf{2,250 Frw}$$

3) James borrowed 400,000 Frw from the bank that charges 5% interest rate per year. Find the interest after 9 months.

4) Samson banked 6,000,000 Frw in a bank that gives 9% interest rate p.a. Calculate the total interest earned after 6 months.

3. <u>Amount (A)</u>

1) Ruzindana deposited 350,000 Frw for 6 months. He earned interest at a rate of 10% Calculate the amount.

P = 350,000 Frw

R = 10%

T = 6 months

$$I = \frac{P \times R \times T}{100 \times 1 \; year} = \frac{350,000 \; Frw \times 10 \times 6 \; months}{100 \times 12 \; months} = \textbf{17,500 Frw}$$

A = P + I = 350,000 Frw + 17,500 Frw

= 367,500 Frw

2) Amanda borrowed 400,000 frw from a bank that charges 15% interest rate per annum. How much did he pay back after $3\frac{1}{2}$ years?

3) Mugenzi banked 250,000 Frw in a bank that offers 8% interest rate p.a in 2 years. Find his amount.

4) Sarah deposited 800,000 Frw in a bank at 15% interest rate p.a. How much money did he find in his account at the start of the third year?

4. <u>Principal or capital (P)</u>

1) A man banked some money in a bank that gives simple interest at a rate of 15% per year. He earned interest of 9,000 Frw in 2 years. How much money did he bank?

$$I = 9{,}000 \text{ Frw}$$
$$R = 15\%$$
$$T = 2 \text{ years}$$
$$P = \frac{I \times 100 \times 1 \, year}{R \times T} = \frac{9{,}000 \, Frw \times 100 \times 1 \, year}{15 \times 2 \, years} = 30{,}000 \text{ Frw}$$

2) How much money did Peter deposit in the bank that offers 10% interest rate in order to get 15,000 Frw as interest in 3 years.

3) What sum of money will yield an interest of 36,000 Frw for 3 months at 10% per year?

4) John made a simple interest of 40,000 Frw in 2 years at interest rate of 5% per year. How much money did John bank?

5) Anita borrowed some money from the bank that charges $2\frac{1}{2}\%$ interest rate for 2 years. If he paid 10,000 Frw as interest, how much money did he pay?

6) Amani banked some money at 8% interest rate p.a. If he got interest of 3,000 Frw, how much money did he bank?

4. Interest rate (R)

1) Calculate the interest rate if 5,000 Frw yielded a simple interest of 1,050 Frw in 3 years.

$$P = 5{,}000 \text{ Frw}$$
$$I = 1{,}050 \text{ Frw}$$
$$T = 3 \text{ years}$$
$$R = \frac{I \times 100 \times 1 \, year}{P \times T} = \frac{1{,}050 \, Frw \times 100 \times 1 \, year}{5{,}000 \, Frw \times 3 \, years} = 7\%$$

2) Zaninka borrowed 60,000 Frw on her account. After 8 months she paid back 64,000 Frw. Calculate the interest rate.

3) George deposited 50,000 Frw on his savings account. At the end of 3 years, the simple interest earned was 15,000 Frw. Calculate the rate of interest.

4) Asimwe deposited 25,000 F which earned interest of 3,000 F in 6 months. What was the interest rate?

5. Time (T)

1) How long will 16,000 Frw take to amount 22,400 Frw at 10% simple interest rate?

$$P = 16{,}000 \text{ Frw}$$
$$A = 22{,}400 \text{ Frw}$$
$$I = A - P = 22{,}400 \text{ Frw} - 16{,}000 \text{ Frw} = 6{,}400 \text{ Frw}$$
$$R = 10\%$$

$$T = \frac{I \times 100 \times 1\ year}{P \times R} = \frac{6,400\ Frw \times 100 \times 1\ year}{16,000\ Frw \times 10} = \textbf{4 years}$$

2) A lady borrowed 25,000 F at 3% and paid interest of 7,500 Frw. Find the time.

3) Eric kept 40,000 Frw in a bank that pays a simple interest rate of 2% per annum. After how long did it amount to 43,600 Frw?

4) A company paid a loan plus interest worth 560,000 Frw after borrowing 420,000 Frw at 10% p.a. For how long did the company use the money?

6. Finding principal given the amount

1) Kamana borrowed some money at 10% interest rate in 2 years. If he paid the amount of 360,000 Frw, how much did he borrow?

A = 360,000 Frw

T = 2 years

R = 10%

Let x = P

$$I = \frac{P \times R \times T}{100 \times 1\ year} = \frac{x \times 10 \times 2\ years}{100 \times 1\ year} = \frac{x}{5}$$

A = P + I

$$360,000 = x + \frac{x}{5}$$

$$\frac{360,000 \times 5}{5} = \frac{5x+x}{5}$$

$$360,000 \times 5 = 5x + x$$

1,800,000 = 6x

6x = 1,800,000

$$\frac{6x}{6} = \frac{1,800,000}{6}$$

x = 300,000 Frw

He borrowed 300,000 Frw

2) Namahire banked some money that amounted to 800,000 Frw in 3 years at 20% interest rate. How much did he bank?

3) Alloys deposited some money in the bank that amounted to 65,000 Frw in 3 years at 10% interest rate. How much money did he deposit?

4) Mugeni banked some money in the bank that offers 15% interest rate in 8 months. If the money amounted to 52,800 frw, how much was the capital?

5) A trader borrowed some money from a bank at an interest rate of 5% per annum. If he paid back 410,000F, find the principal.

TOPIC 12: <u>FRACTIONS</u>

1. <u>The notion of a fraction</u>

Definition: A fraction is a part of the whole.

Study the following and state the fractions representing the shaded part.

 $= \dfrac{2}{6}$ $= \dfrac{4}{10}$ $= \dfrac{1}{3}$
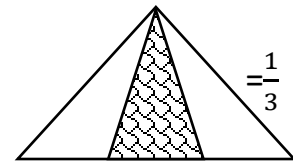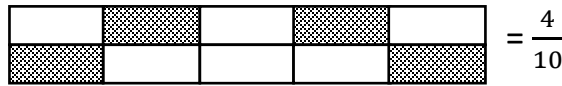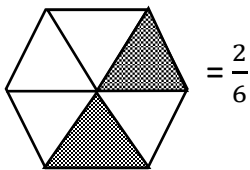
2. The parts of a fraction

A fraction has three main parts: **- Numerator**

                                  - Denominator

                                  - Fraction bar

numerator

eg: $\dfrac{4}{9}$ → fraction bar
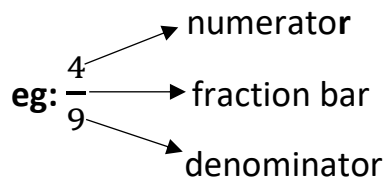
→ denominator

3. Types of a fraction

There are three types of fractions: **- Proper fractions**

                                  - Improper fractions

                                  - Mixed fractions

a) Proper fractions

A proper fraction is a fraction whose numerator is smaller than the denominator.

eg: $\dfrac{1}{5} ; \dfrac{7}{20} ; \dfrac{5}{9} ; \dfrac{3}{14} ;$ ………

b) Improper fractions

An improper fraction is a fraction is whose numerator is bigger than the denominator.

eg: $\dfrac{4}{3} ; \dfrac{9}{7} ; \dfrac{12}{11} ; \dfrac{15}{4} ;$ ………

c) Mixed fractions

A mixed fraction is a fraction made up of a whole number and a fraction.

eg: $1\dfrac{1}{2} ; 2\dfrac{3}{5} ; 4\dfrac{5}{11} ;$ ………

4. Writing and writing fractions

When reading a fraction, the numerator is read as a **cardinal number** while the denominator is read as an **ordinal number.**

Write the following fractions in words:

a) $\dfrac{2}{5}$ = **two fifths**　　　　　　　c) $\dfrac{7}{9}$ = **seven ninths**

b) $\dfrac{3}{8}$ = **three eights**　　　　　　d) $\dfrac{1}{10}$ = **a tenth**

e) $\frac{5}{12}$ = **five twelves**   h) $\frac{1}{3}$ = **a third**

f) $\frac{7}{20}$ = **seven twentieths**

g) $\frac{11}{50}$ = **eleven fiftieths**

Exception: * $\frac{1}{2}$ = **a half**   * $\frac{3}{4}$ = **three quarters**

* $\frac{1}{4}$ = **a quarter**   * $\frac{2}{4}$ = **two quarters**

5. Changing improper fractions to mixed fractions

1) Change $\frac{4}{3}$ into a mixed fraction.

$\frac{4}{3}$ = **1$\frac{1}{3}$**

2) Express $\frac{9}{5}$ as a mixed fraction.

$\frac{9}{5}$ = 1$\frac{4}{5}$

6. Changing mixed fractions into improper fractions

1) Change 1$\frac{3}{5}$ into an improper fraction.

1$\frac{3}{5}$ = $\frac{8}{5}$

2) Express 4$\frac{1}{6}$ as a mixed fraction.

4$\frac{1}{6}$ = $\frac{25}{6}$

7. Expressing fractions as decimals

Express the following fractions as decimals:

a) $\frac{2}{5}$ = **0.4**

b) $\frac{1}{2}$ = **0.25**

c) $\frac{3}{4}$ = **0.75**

d) $\frac{5}{8}$ = **0.625**

e) 2$\frac{1}{5}$ = **2.2**

8. Changing decimals into fractions

Express the following decimals as fractions in its lowest terms.

a) 0.4 = $\frac{4}{10}$

    = $\frac{2}{5}$

b) 3.5 = $\frac{35}{10}$

$$= \frac{7}{2} = 3\frac{1}{2}$$

c) $0.053 = \frac{53}{1000}$

d) $0.75 = \frac{75}{100}$

$$= \frac{3}{4}$$

e) $2.25 = \frac{225}{100}$

$$= 2\frac{1}{4}$$

9. Expressing fractions as ratios

1) Express $\frac{3}{5}$ as a ratio.

$\Longrightarrow$ **3:5**

2) Express $1\frac{4}{9}$ as a ratio.

$$1\frac{4}{9} = \frac{13}{9}$$

$\Longrightarrow$ **13:9**

10. Expressing ratios as a fraction

1) Express 2:5 as a fraction.

Fraction $= \frac{2}{5}$

2) Express 9:2 as a fraction.

Fraction $= \frac{9}{2} = 4\frac{1}{2}$

11. Expressing fractions as percentages

Express the following fractions as a percentage:

a) $\frac{18}{100} = \frac{18}{100} \times$ **100%**

$\quad\quad$ **= 18%**

b) $\frac{1}{2} = \frac{1}{2} \times$ **100%**

$\quad\quad$ **= 50%**

c) $\frac{7}{10} = \frac{7}{10} \times$ **100%**

$\quad\quad$ **= 70%**

d) $2\frac{1}{4} = \frac{9}{4} \times$ **100%**

$\quad\quad$ **=225%**

12. Expressing fractions as percentages

Express the following percentages as fractions in their lowest terms:

a) $65\% = \frac{65}{100}$

$$= \frac{13}{20}$$

b) $2\frac{1}{4}\% = \frac{9}{4}\% = \frac{\frac{9}{4}}{100}\%$

$$= \frac{9}{4} \div 100$$

$$= \frac{9}{4} \times \frac{1}{100} = \frac{9}{400}$$

c) $150\% = \frac{150}{100} = \frac{3}{2}$

$$= 1\frac{1}{2}$$

13. Express quantities as fractions

1) Thomas got 60 marks out of 80 in an exam. Express his marks as a fraction.

$$\textbf{Fraction} = \frac{60}{80} = \frac{3}{4}$$

2) There were 50 desks in a class. 30 desks are broken. What fraction represents the broken desks?

$$\textbf{Fraction} = \frac{30}{50} = \frac{3}{5}$$

14. Expressing quantities as fractions, then as percentages

1) Mutoni got 80 marks out of 100. Express her marks as a percentage.

$$\textbf{Fraction} = \frac{80}{100} = \frac{4}{5}$$

$$\% = \frac{4}{5} \times 100\% = 80\%$$

2) There are 20 boys and 30 girls in a class. Find the percentage of boys.

$$\textbf{Fraction} = \frac{20}{50} = \frac{2}{5}$$

$$\% = \frac{2}{5} \times 100\% = 40\%$$

3) In a box there are 50 pens. If 40 are blues. Find the percentage of black pens.

4) Kabera had 20 cows and sold 5 cows. What is the percentage representing the sold cows?

5) In a class of 20 pupils, there are 8 girls. Find the percentage of boys.

15. Comparing fractions

- When two fractions have the same numerators, the one having a small denominator is a greater one.

eg: * $\frac{3}{5} < \frac{3}{4}$

* $\dfrac{8}{15} > \dfrac{8}{30}$

* $\dfrac{6}{9} < \dfrac{6}{7}$

- When two fractions have the same denominators the one having a big numerator is a greater one.

eg: * $\dfrac{3}{5} < \dfrac{4}{5}$

* $\dfrac{6}{8} > \dfrac{5}{8}$

* $\dfrac{9}{10} > \dfrac{6}{10}$

- For other fractions, before comparing them, first change them into decimal numbers.

eg: * $\dfrac{3}{4} > \dfrac{2}{5}$

    0.75   0.4

* $\dfrac{1}{4} > \dfrac{1}{5}$

    0.25   0.2

16. Ordering fractions

1) Arrange the following fractions from the largest to the smallest.

$\dfrac{5}{12} ; \dfrac{1}{12} ; \dfrac{7}{12} ; \dfrac{11}{12}$

Solution: $\dfrac{11}{12} ; \dfrac{7}{12} ; \dfrac{5}{12} ; \dfrac{1}{12}$

2) Re-arrange the following fractions is ascending order.

$\dfrac{5}{8} ; \dfrac{5}{11} ; \dfrac{5}{9} ; \dfrac{5}{13} ; \dfrac{5}{6}$

Solution: $\dfrac{5}{13} ; \dfrac{5}{11} ; \dfrac{5}{9} ; \dfrac{5}{8} ; \dfrac{5}{6}$

3) Arrange the given fractions in ascending order.

$\dfrac{2}{3} ; \dfrac{3}{4} ; \dfrac{1}{2} ; \dfrac{5}{6}$

Solution: $\dfrac{2}{3} = 0.66..$ $\dfrac{3}{4} = 0.75$ $\dfrac{1}{2} = 0.5$ $\dfrac{5}{6} = 0.833$

$\Longrightarrow \dfrac{1}{2} ; \dfrac{2}{3} ; \dfrac{3}{4} ; \dfrac{5}{6}$

4) Arrange the following in descending order.

$0.45 ; \dfrac{1}{3} ; 0.64 ; \dfrac{5}{6}$

17. Reciprocal of fractions

A reciprocal is a number that is related to another number by the fact that when the two are multiplied their product is 1.

eg: - The reciprocal of $\frac{1}{4}$ is $\frac{4}{1}$ or 4

- The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$

- The reciprocal of $\frac{5}{8}$ is $\frac{8}{5}$

18. The complement of a fraction

A complement of a fraction is a fraction that can be added to that one to make a whole.

eg: - The complement of $\frac{1}{4}$ is $\frac{3}{4}$

- The complement of $\frac{2}{3}$ is $\frac{1}{3}$

19. Equivalent fractions

Equivalent fractions are fractions which are equal.

Examples

1) Write any two equivalent fractions to $\frac{2}{3}$

- $\frac{2 \times 2}{3 \times 2} = \frac{4}{6}$

- $\frac{2 \times 3}{3 \times 3} = \frac{6}{9}$

2) Give any three fractions equivalent to $\frac{1}{2}$

- $\frac{1 \times 2}{2 \times 2} = \frac{2}{4}$

- $\frac{1 \times 3}{2 \times 3} = \frac{3}{6}$

- $\frac{1 \times 5}{2 \times 5} = \frac{5}{10}$

- $\frac{1 \times 7}{2 \times 7} = \frac{7}{14}$

3) Find the equivalent fraction to $\frac{3}{5}$ whose numerator is 9.

$\frac{3 \times 3}{5 \times 3} = \frac{9}{15}$

The fraction is $\frac{9}{15}$

4) Find the fraction equivalent to $\frac{4}{7}$ whose denominator is 28.

$\frac{4 \times 4}{7 \times 4} = \frac{16}{28}$

The fraction is $\frac{16}{28}$

5) Calculate the equivalent fraction to $\frac{2}{5}$ whose sum of terms is 21.

Sum = 21

Sum of ratios = 2 + 5 = 7

1 ratio = 21 ÷ 7 = 3

Numerator = 2 x 3 = 6

Denominator = 5 x 3 = 15

The fraction is $\frac{6}{15}$

6) Find the equivalent fraction to $\frac{4}{7}$ whose difference of terms is 6.

Difference = 6

Difference of ratios = 7 – 4 = 3

1 ratio = 6 ÷ 3 = 2

Numerator = 4 x 2 = 8

Denominator = 7 x 2 = 14

The fraction is $\frac{8}{14}$

20. Reducing fractions

Simplify completely:

a) $\frac{60}{70} = \frac{\cancel{2} \times 2 \times 3 \times \cancel{5}}{\cancel{2} \times \cancel{5} \times 7} = \frac{6}{7}$

b) $\frac{45}{75} = \frac{\cancel{3} \times 3 \times \cancel{5}}{\cancel{3} \times \cancel{5} \times 5} = \frac{3}{5}$

c) $\frac{27}{81} = \frac{\cancel{3} \times \cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{3} \times \cancel{3} \times 3} = \frac{1}{3}$

Exercises

Simplify completely:

a) $\frac{36}{48}$

b) $\frac{72}{108}$

c) $\frac{90}{120}$

d) $\frac{96}{144}$

e) $\frac{180}{420}$

f) $\frac{100}{125}$

g) $\frac{75}{125}$

h) $\frac{90}{135}$

i) $\frac{60}{80}$

21. Operations in fractions

a) Addition and subtraction

When adding or subtracting two fractions with the same denominators, add or subtract the numerators and keep one denominator.

Examples

Work out:

a) $\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$

b) $\frac{6}{7} - \frac{2}{7} = \frac{4}{7}$

c) $\frac{4}{9} + \frac{7}{9} = \frac{11}{9} = 1\frac{2}{9}$

d) $2\frac{4}{5} - 1\frac{1}{5} = \frac{14}{5} - \frac{6}{5} = \frac{8}{5} = 1\frac{3}{5}$

e) $2\frac{3}{8} + 1\frac{1}{8} = \frac{19}{8} + \frac{9}{8} = \frac{28}{8} = 3\frac{4}{8} = 3\frac{1}{2}$

When adding or subtracting fractions with different denominators, first put them on the common denominator.

Examples

Work out:

a) $\frac{1}{2} + \frac{1}{4} = \frac{2+1}{4} = \frac{3}{4}$

b) $\frac{5}{6} - \frac{3}{8} = \frac{20-9}{24} = \frac{11}{24}$

c) $\frac{1}{4} + \frac{1}{6} = \frac{3+2}{12} = \frac{5}{12}$

d) $3\frac{1}{5} - 1\frac{1}{2} = \frac{16}{5} - \frac{3}{2} = \frac{32-15}{10} = \frac{17}{10} = 1\frac{7}{10}$

e) $2\frac{1}{2} + \frac{1}{3} = \frac{5}{2} + \frac{1}{3} = \frac{15+2}{6} = \frac{17}{6} = 2\frac{5}{6}$

f) $\frac{1}{5} + 2 = \frac{1+10}{5} = \frac{11}{5} = 2\frac{1}{5}$

g) $3 - \frac{1}{4} = \frac{12-1}{4} = \frac{11}{4} = 2\frac{3}{4}$

Addition and subtraction together

Work out:

a) $\frac{1}{4} - \frac{3}{4} + \frac{2}{3} = \left(\frac{1}{4} + \frac{2}{3}\right) - \frac{3}{4}$

$$= \left(\frac{3+8}{12}\right) - \frac{3}{4}$$

$$= \frac{11}{12} - \frac{3}{4} = \frac{11-9}{12}$$

$$= \frac{2}{12} = \frac{1}{6}$$

Exercises

Simplify:

b) $\frac{3}{8} - \frac{1}{2} + \frac{3}{4} =$

c) $\frac{3}{5} + \frac{1}{10} - \frac{3}{5} =$

d) $2 + \frac{1}{3} - \frac{1}{4} =$

e) $\frac{1}{2} - \frac{5}{6} + \frac{7}{8} =$

f) $2\frac{5}{8} - 3\frac{1}{3} + 4\frac{3}{4} =$

g) $\frac{7}{12} + \frac{1}{2} - \frac{5}{6} =$

h) $\frac{5}{6} - \frac{1}{2} + \frac{2}{3} - \frac{1}{4} =$

i) $3\frac{2}{3} - 1\frac{5}{6} + 2\frac{5}{8} =$

j) $\frac{1}{3} - \frac{3}{5} + \frac{7}{10} =$

k) $\frac{1}{2} - \frac{2}{5} + \frac{3}{10} =$

l) $\frac{1}{3} + \frac{1}{5} - \frac{1}{4}$

m) $\frac{7}{10} - \frac{7}{15} + \frac{5}{6} =$

n) $\frac{5}{6} - \frac{1}{3} + \frac{1}{2} - \frac{1}{5} =$

Word problem involving addition and subtraction

1) Ana had $\frac{1}{2}$ glass full of water and used $\frac{1}{3}$ of it to take medicine. What fraction of water remained?

2) In Amahoro stadium $\frac{1}{6}$ of the seats is filled by women, $\frac{3}{5}$ by men and $\frac{1}{15}$ by children. What fraction of the stadium is occupied?

3) $\frac{1}{3}$ of the meeting room is filled by children, $\frac{1}{5}$ by men and $\frac{2}{5}$ by women. What fraction of the seats in the meeting room are occupied?

4) A boy had a jerry can full of water, he used $\frac{13}{20}$. What fraction remained?

5) A tank was $\frac{2}{3}$ full of water and after it rained, the tank was $\frac{11}{12}$ full. What fraction was added?

b) Multiplication

When multiplying a fraction by a fraction, multiply both numerators separately and then denominators separately.

Examples

Multiply:

a) $3 \times \frac{1}{4} = \frac{3}{4}$

b) $\frac{1}{5} \times \frac{2}{3} = \frac{2}{15}$

c) $1\frac{1}{2} \times 2\frac{1}{6} = \frac{3}{2} \times \frac{13}{6} = \frac{39}{12} = 3\frac{3}{12} = 3\frac{1}{4}$

Finding a fraction of a whole number

1) Find $\frac{3}{4}$ of 200 kg.

$$\frac{3}{4} \text{ of 200 kg} = \frac{3}{4} \times 200kg = 150 \text{ kg}$$

2) What is $\frac{2}{5}$ of 400m + 100m?

3) Calculate $\frac{2}{3}$ of 3000g and express the answer in kg.

4) Find 0.25 of 3,000 Frw.

5) Ben had 10,000 Frw. He gave $\frac{2}{5}$ of t to Gavin. How much money did Gavin get?

6) James, David and Tite shared 40,000 Frw. James got $\frac{1}{4}$, David got $\frac{5}{8}$ and Tite got $\frac{1}{8}$. How much money did each get?

Simplification

Simplify completely:

a) $\frac{36 \times 27 \times 80}{54 \times 12 \times 160} = \frac{\cancel{2} \times \cancel{2} \times \cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{3} \times 3 \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{5}}{\cancel{2} \times \cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{2} \times \cancel{2} \times \cancel{3} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times 2 \times 2 \times \cancel{5}} =$

b) $\frac{18 \times 32 \times 21}{12 \times 64 \times 42} =$

c) $\frac{30 \times 135 \times 81}{27 \times 40 \times 45} =$

d) $\frac{15 \times 9 \times 42}{27 \times 56 \times 45} =$

d) Division

When dividing a fraction by another fraction, multiply the first fraction by the reciprocal of the second fraction.

Examples

Work out:

a) $\frac{3}{4} \div \frac{4}{5} = \frac{3}{4} \times \frac{5}{4} = \frac{15}{16}$

b) $2\frac{1}{3} \div 1\frac{3}{4} =$

c) $\frac{2}{5} \div 2 =$

d) $\frac{3\frac{1}{3}}{1\frac{3}{4}} =$

e) $54 \div 1\frac{1}{5} =$

More about fractions

When working out fractions involving all operations, we follow the rule of **BODMAS**

B = Brackets **O** = Of **D** = Division **M** = Multiplication **A** = Addition **S** = Subtraction

Examples

Work out:

a) $\frac{2}{5} - \frac{1}{4} \times \frac{3}{4} \div \frac{5}{8} = \frac{2}{5} - \frac{1}{4} \times \left(\frac{3}{4} \div \frac{5}{8}\right)$

$$= \frac{2}{5} - \frac{1}{4} \times \left(\frac{3}{4} \times \frac{8}{5}\right)$$

$$= \frac{2}{5} - \frac{1}{4} \times \left(\frac{3}{4^1} \times \frac{8^2}{5}\right)$$

$$= \frac{2}{5} - \left(\frac{1}{4^2} \times \frac{6^3}{5}\right)$$

$$= \frac{2}{5} - \frac{3}{10}$$

$$= \frac{4-3}{10} = \frac{1}{10}$$

b) $\frac{1}{2} + \frac{2}{3} \times \frac{3}{5} \div \frac{6}{7} - \frac{5}{6} =$

c) $\frac{5}{6} - \frac{3}{8} \div \frac{3}{4} + \frac{5}{8} \times \frac{2}{15} =$

d) $\frac{4}{10} \times 1\frac{1}{4} + \frac{4}{5} \div 1\frac{1}{5} =$

e) $0.25 + 1\frac{1}{3} \ of \ \left(\frac{1}{2} - \frac{1}{7}\right) \div 1\frac{3}{7} =$

Exercises

Simplify completely:

1) $\dfrac{\frac{1}{6}+0.5 \times \frac{2}{9} \div \left(\frac{3}{15} \ of \ 1\frac{2}{3}\right)}{\frac{2}{3} - \left(\frac{1}{3}+\frac{1}{2}\right) \times \frac{1}{5}} =$

5) $\dfrac{3\frac{1}{4} - \left(\frac{1}{2}+\frac{1}{4}\right) \div 0.25}{0.5 \div 1\frac{1}{3}+\frac{10}{12} \ of \ \frac{3}{10}} =$

2) $\dfrac{1\frac{1}{2} \times \frac{8}{10} \ of \frac{5}{6}-\frac{1}{4} \div \frac{2}{4}}{\frac{4}{6} \times \left(\frac{1}{4}+\frac{1}{6}\right) \div 1\frac{2}{3}} =$

6) $\dfrac{0.2+1\frac{1}{4} \div \frac{6}{12} \ of \ 1\frac{2}{3}}{\frac{2}{5}+\left(\frac{1}{2}-\frac{1}{4}\right) \div \frac{2}{4}} =$

3) $\dfrac{\frac{16}{0.25}+2\frac{1}{2} \div 0.5+3\frac{2}{3} \div \frac{1}{3}}{2\frac{2}{5} \times (1\frac{2}{3} \div \frac{1}{2})} =$

7) $\dfrac{1\frac{1}{2} \times \frac{1}{6}+0.5 \div \frac{3}{4}}{\frac{1}{4} \times 2\frac{2}{3}-\frac{5}{6} of \frac{3}{5}} =$

4) $\dfrac{3\frac{1}{4} - \left(\frac{1}{2}+\frac{1}{4}\right) \div 0.25}{0.5 \div 1\frac{1}{3}+\frac{10}{12} \ of \ \frac{3}{10}} =$

8) $\dfrac{\frac{4}{6} + 1\frac{1}{3} \times \frac{3}{5} - \left(\frac{5}{7} \times \frac{21}{25} - \frac{1}{3}\right)}{\frac{3}{4} \div 1\frac{1}{4} + \frac{2}{10}} =$

9) $\dfrac{\frac{3}{0.5} \times 1\frac{1}{2} + 0.75 \; of \; 2\frac{2}{3}}{4\frac{4}{5} \times \left(\frac{1}{2} + \frac{1}{3}\right) + \frac{1}{2} \div \frac{1}{10}} =$

10) $\dfrac{\frac{1}{4} + 1\frac{1}{3} \; of \; \left(\frac{1}{2} - \frac{1}{7}\right) \div 1\frac{3}{7}}{\frac{4}{10} \times 1\frac{1}{4} + \frac{4}{5} \div 1\frac{1}{5}} =$

11) $\dfrac{\frac{1}{6} + 0.5 \times \frac{2}{9} \div \frac{3}{15} \; of \; 1\frac{2}{3}}{\frac{2}{3} - \left(\frac{1}{3} + \frac{1}{2}\right) \times \frac{1}{5}} =$

15) $\dfrac{\frac{4}{6} + 1\frac{1}{3} \times \frac{3}{5} - \left(\frac{5}{7} \times \frac{21}{25} - \frac{1}{3}\right)}{\frac{3}{4} \div 1\frac{1}{4} + \frac{2}{10}} =$

16) $\dfrac{\frac{24}{0.25} + 3\frac{5}{10} \div 0.5 + \frac{24}{0.75}}{\frac{2}{7} \div \left(\frac{3}{5} \times \frac{1}{6} \times \frac{8}{9}\right)} =$

17) $\dfrac{\frac{48}{0.25} + 3.5 \div \frac{1}{2} + \frac{24}{0.75}}{\frac{1}{7} \div \left(\frac{6}{10} \times \frac{1}{6} \times \frac{16}{14}\right)} =$

18) $\dfrac{\frac{72 \times 5}{9} + \frac{96 \times 7}{12}}{0.96} \div \frac{1}{5} =$

12) $\dfrac{1\frac{1}{2} - \left(\frac{1}{3} + \frac{1}{6}\right) \div 0.75}{0.2 \div 1\frac{1}{5} + \frac{8}{10} \; of \; \frac{5}{8}} =$

13) $\dfrac{\frac{4}{5} \times 0.25 + 2\frac{2}{3} \; of \; \frac{1}{4} \div 1\frac{1}{3}}{\left(\frac{2}{4} - \frac{1}{6}\right) \times \frac{2}{6} \div 1\frac{1}{3}} =$

14) $\dfrac{1\frac{3}{4} \div 2\frac{5}{8} + 1\frac{1}{2} \; of \; \frac{10}{12} - \frac{5}{10}}{\frac{2}{3} \times 0.25 + 5\frac{1}{3} \times \frac{1}{2}} =$

19) $\dfrac{\left(3\frac{1}{2} \times 1\frac{1}{4}\right) \div \left(2\frac{1}{2} - 1\frac{3}{4}\right)}{2.3 \div 4.6} =$

20) $\dfrac{\frac{1}{5} \times \left(6\frac{3}{4} - 4.75\right) \times \left(3.875 - 2\frac{3}{8}\right)}{(1.5 \times 1.5) \div 2\frac{1}{2}} =$

21) $\dfrac{\frac{14}{28} \div \frac{10}{14} \times 3.25 - \frac{4}{7} \times 3\frac{3}{16} - \frac{10}{28}}{\frac{3}{16} - 0.75 \times \frac{1}{6}} =$

22) $\dfrac{48 \times \frac{2}{4} \times \frac{5}{8} \times \frac{2}{5} + 48 \times \frac{1}{6}}{14 \times \frac{1}{2}} =$

Word problems involving operations (Part I)

1) Anna had a $\frac{1}{2}$ glass full of water and used $\frac{1}{3}$ of it to take medicine. What fraction of water was left?

2) John ate $\frac{1}{4}$ of the sugar cane in the morning, $\frac{1}{2}$ at lunch time. What fraction of the used cane was eaten?

3) Two boxes weigh $1\frac{1}{2}$kg and $3\frac{3}{4}$kg. What is their total weight?

4) A tank was $\frac{2}{3}$ full of water and after it rained, the tank was $\frac{11}{12}$ full. What fraction was added?

5) Subtract $3\frac{1}{4}$ from $7\frac{3}{5}$

6) The product of two numbers is $\frac{2}{3}$, one number is $\frac{5}{4}$. Find the other number.

7) A kilometer is about $\frac{5}{8}$ of a mile. How many kilometres are in $\frac{3}{4}$ of a mile?

8) How many plots of land each $\frac{2}{5}m^2$ can be obtained from a big piece of land of $28m^2$?

9) How many bottles each $\frac{1}{4}$ a litre of milk can be filled from a jerry can of 4 litres?

10) How many packets of $2\frac{1}{4}$kg are there in 8kg of sugar?

Word problems involving operations (Part II)

1) $\frac{1}{3}$ of the meeting room is filled by children, $\frac{1}{5}$ by men and $\frac{2}{5}$ by women. What fraction of the seats in the meeting room is occupied?

2) At Gatuna primary school $\frac{1}{3}$ of the pupils like football, $\frac{3}{12}$ like volleyball and $\frac{1}{8}$ like swimming.
   - a) What fraction of the pupils like games?
   - b) What fraction of the pupils do not like games?
   - c) If the school has 4,800 pupils, how many pupils like games altogether?
   - d) How many pupils like football?
   - e) How many pupils like football?
   - f) How many pupils like swimming?

3) In Amahoro stadium $\frac{1}{6}$ of the seats are filled by women, $\frac{3}{5}$ by men and $\frac{1}{15}$ by children.
   - g) What fraction of the stadium is occupied?
   - h) What fraction of the stadium is unoccupied?
   - i) If the stadium has 12,000 seats, how many people are there altogether?
   - j) How many women are there?
   - k) How many men are there?
   - l) How many children are there?

4) In a conference hall, $\frac{2}{6}$ of seats are filled by women, $\frac{1}{5}$ by men and $\frac{1}{3}$ by children.
   - (a) What fraction of the conference hall is occupied?
   - (b) What fraction of the conference hall is not occupied?
   - (c) How many people are in the conference hall if the whole conference room contains 9000 seats?
   - (d) Calculate the number of men who are present.
   - (e) Calculate the number of women who are present.
   - (f) Calculate the number of children who are present.

Application of fractions (Part I)

1) Peter gave $\frac{2}{5}$ of his money to Paul. If he had 40,000 Frw, how much money did Paul receive?

Total = $\frac{5}{5}$

Paul = $\frac{2}{5}$

Remainder = $\frac{5}{5} - \frac{2}{5} = \frac{3}{5}$

$\begin{matrix} \frac{5}{5} \\ \frac{2}{5} \end{matrix}$ ⟶ $\frac{40,000\ Frw \times 2}{5} = \mathbf{16,000\ Frw}$

2) John spent $\frac{5}{8}$ of the money he had for clothes. If he had 32,000 Frw, how much money did he remain with?

Total = $\frac{8}{8}$

Clothes = $\frac{5}{8}$

Remainder = $\frac{8}{8} - \frac{5}{8} = \frac{3}{8}$

$\begin{matrix} \frac{8}{8} \\ \frac{3}{8} \end{matrix}$ ⟶ $\frac{32,000\ Frw \times 3}{8} = \mathbf{12,000\ Frw}$

3) Ana used $\frac{3}{7}$ of her salary for buying food. If she paid 9,000 Frw for food, how much much is her salary?

4) John spent $\frac{5}{8}$ of the money he had for clothes. If he had 32,000 Frw, how much money did he remain with?

5) A man had 20 kg of rice. He gave $\frac{1}{4}$ to his uncle. How many kg did he remain with?

6) A girl used $\frac{7}{11}$ of her money on clothes. If she remained with 12,000 Frw, how much money did she pay for clothes?

7) Ana got $\frac{3}{4}$ of her salary. If she got 21,000 Frw, what was his salary?

8) A man received $\frac{9}{10}$ of his salary. If his salary was 20,000 Frw, how much money did he receive?

9) John used $\frac{3}{5}$ of his money for buying shoes. If he remained with 10,000 Frw, what was his salary?

Application of fractions (Part I)

1) A woman spent $\frac{1}{3}$ of her money on treatment, $\frac{1}{2}$ on food. If she remained with 6,000 F, how much money did she have?

$$\text{Total} = \frac{3}{3}$$

$$\text{Treatment} = \frac{1}{3}$$

$$\text{Remainder} = \frac{3}{3} - \frac{1}{3} = \frac{2}{3}$$

$$\text{Rent} = \frac{1}{2}$$

$$\text{Remainder} = \frac{2}{3} - \frac{1}{2} = \frac{4-3}{6} = \frac{1}{6}$$

$$\frac{1}{6}$$
$$\frac{6}{6}$$ → $$\frac{6{,}000 \; Frw \times 6}{1} = 36{,}000 \; Frw$$

2) A man spent $\frac{1}{4}$ of his salary on food, $\frac{2}{5}$ of the remainder on clothes and saved the 18,000 Frw. Find the man's salary.

$$\text{Total} = \frac{4}{4}$$

$$\text{Food} = \frac{1}{4} \quad 10000$$

$$\text{Remainder} = \frac{4}{4} - \frac{1}{4} = \frac{3}{4}$$

$$\text{Clothes} = \frac{2}{5} \text{ of } \frac{3}{4} = \frac{2}{5} \times \frac{3}{4} = \frac{3}{10}$$

$$\text{Remainder} = \frac{3}{4} - \frac{3}{10} = \frac{15-6}{20} = \frac{9}{20}$$

$$\frac{9}{20}$$
$$\frac{20}{20}$$ → $$\frac{18{,}000 \; Frw \times 20}{9} = 40{,}000 \; Frw$$

3) $\frac{1}{3}$ of the wire is painted black, $\frac{2}{5}$ of the remainder is painted green, the rest of 20 metre wire is red. How long was the wire?

4) Joan did $\frac{1}{2}$ of her homework on Friday and $\frac{1}{4}$ of the remainder on Sunday. If she completed 24 numbers on Sunday. How many numbers were given to her as homework?

5) Jasmine had 300,000 Frw in her purse. She gave $\frac{3}{10}$ of it to Jane, $\frac{1}{5}$ to Julian and saved the rest.
   a) What fraction of the money did she save?
   b) How much money did Jasmine save?

6) Three men shared 800 kg of beans. Ali got $\frac{1}{4}$ of the beans, Moses got $\frac{3}{8}$ and Katto got the remaining beans.
   a) Calculate the fraction Katto got.
   b) Work out the amount of beans in kilogram each got.

7) Moses spent $\frac{1}{4}$ of his money on food. He also spent $\frac{1}{3}$ of the remaining money on transport. He was left with 12,000 Frw. How much money did he have originally?

8) A painter painted $\frac{1}{5}$ of the room with white colour and $\frac{4}{10}$ of it with blue colour. He then painted the remaining part with 5 litres of green colour.

    a) What part was painted green?

    b) Work out the amount of paint the painter used to paint the entire classroom.

9) Jim ate $\frac{2}{5}$ of a pancake and Joan ate $\frac{1}{3}$ of the pancake. The rest was eaten by Peter.

    a) What fraction did Peter eat?

    b) If peter's part weighed 20 grams, what was the total weight of the pancake?

10) A, B and C owned a business. A's share is $\frac{2}{11}$ of the total share capital, B's share is $\frac{5}{11}$ of the total shares.

    a) What fraction of the shares did C own?

    b) If C's share was 56,000 Frw, what was their total share capital?

11) John, Andrew and Moses shared some money. John got $\frac{2}{15}$, Andrew got $\frac{7}{15}$

    a) What fraction did Moses get?

    b) If Moses got 12,000 Frw, what amount of money did they share?

Application of fractions (Part II)

1) Tap A can fill a tank in 6 minutes and tap B can fill the same tank in 3 minutes. How long will both taps take to fill the tank if they are opened at the same time?

Tap A	**Tap B**
6 min ⟶ **to fill**	**3 min** ⟶ **to fill**
1 min ⟶ $\frac{1}{6}$	**1 min** ⟶ $\frac{1}{3}$

Both taps in 1 min $= \frac{1}{6} + \frac{1}{3} = \frac{1+2}{6} = \frac{3}{6} = \frac{1}{2}$

$\frac{1}{2} \longrightarrow \frac{1\ min \times 2}{1} = 2\ minutes$

$\frac{2}{2}$

2) A tank ha two taps. The first fills it in 3 minutes while the second draws water from it in 4 minutes. How long will it take to fill the tank if both taps are let open?

1ˢᵗ tap	**2ⁿᵈ tap**
3 min ⟶ **to fill**	**4 min** ⟶ **to empty**
1 min ⟶ $\frac{1}{3}$	**1 min** ⟶ $\frac{1}{4}$

Both taps in 1 min $= \frac{1}{3} - \frac{1}{4} = \frac{4-3}{12} = \frac{1}{12}$

$$\frac{1}{12} \qquad \frac{1 \, min \times 12}{1} = 12 \, minutes$$

$$\frac{12}{12}$$

3) Tap A takes 9 minutes to fill the tank, tap b takes 12 minutes but tap C takes only 6 minutes to empty the tank. How long will it take to fill the tank if all the taps are open?

4) Tap A takes 9 minutes to fill the tank. Tap B takes 12 minutes and tap C takes 18 minutes. How long will A, B and C take to fill the tank if opened together?

5) A tank has two taps, one fills $\frac{1}{10}$ in one minute and another tap fills $\frac{1}{15}$ of the tank in one minute. How long will both taps take to fill the tank?

6) Akide can dig a garden in 8 days and Bernard can dig it in 10 days.
   a) What fraction of the garden can they dig in 1 day if both work together?
   b) What fraction of the garden is left after both have been working for 4 days?

7) Tap A fills a tank in 2hours, tap B in 3 hours while tap C empties the tank in 6 hours. If all taps are open at the same time, after how long will the tank be full?

8) The first tap fills a tank in 4 minutes while the second empties it in 6 minutes. If both taps are opened at the same time, after how long will the tank be full?

9) A tank has three taps. Tap A fills it in 4 minutes, tap B fills it in 6 minutes while tap C empties the tank in 12 minutes. If all taps are opened at the same time, after how long will the tank e full?

TOPIC 13: <u>DECIMALS</u>

1) <u>The notion of a decimal</u>

A decimal number is any number which contains a **decimal point.**

A decimal number also known as a **decimal fraction** is another way of expressing a fraction.

Eg:
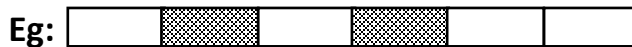
Two fifths $= \frac{2}{5} = 0.4$

2) <u>Parts of a decimal number</u>

A decimal numeral consists of three parts: - **whole number**

- **decimal part**

- **decimal point**

Eg: 0.5

→ **Decimal part**

→ **Decimal point**
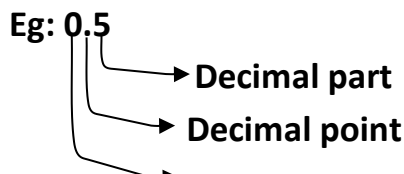
Whole number

3) Place values

1) Write the place value of each digit in the number 23.0145

23.0145

- →**Ten thousandths**
- →**Thousandths**
- →**Hundredths**
- →**Tenths**
- →**Ones**
- →**Tens**
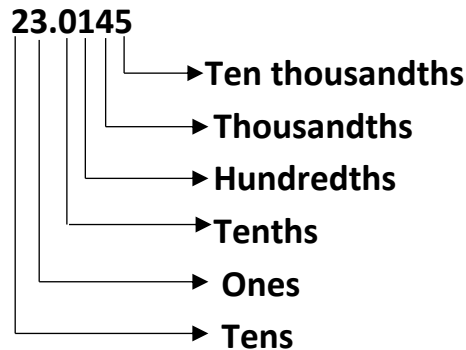
2) What is the rank of 6 in the number 0.2469?

3) What is the position of 4 in the number 726.9401?

4) Values

1) Find the value of each digit in the number 23.0145.

23.0145

- → **5 x 0.0001 = 0.0005**
- → **4 x 001 = 0.004**
- → **1 x 0.01 = 0.01**
- → **0 x 0.1 = 0**
- → **3 x 1 = 3**
- → **2 x 10 = 20**
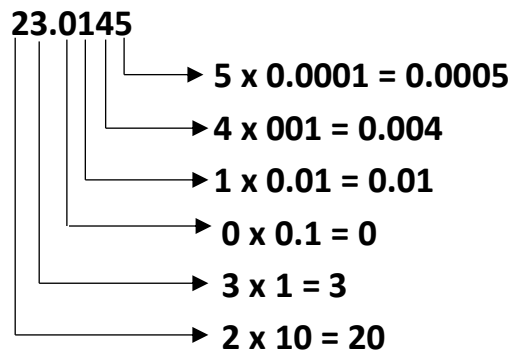
2) What is the value of 9 in the number 0.00249?

3) What are the place value and the value of 3 in 45.936?

4) What is the rank of 4 and the lace value of 7 in the number 2483.9075?

5) Reading and writing decimal numbers

Write the following numbers in words:

a) 0.4 = **Four tenths**

b) 2.15 = **Two hundredths**

c) 3.142 = **Three and one hundred forty to thousandths**

d) 0.0126 = **one hundred twenty six ten thousandths**

e) 32.04 = **Thirty two and four hundredths**

f) 5.8 = **Five and eight tenths**

g) 0.26 = **Twenty six hundredths**

h) 573.075 = **Five hundred seventy three and seventy five thousandths**

6) Comparing decimal numbers

When comparing decimal numbers, first compare their whole parts.

Examples

Compare the following numbers:

a) $21.4 > 7.1498$

b) $6.1465 < 8.1$

c) $2.01 < 3.1$

d) $15.2 > 9.64854$

When whole parts are the same, compare decimal parts digits by digits

a) $2.4 > 2.3987$

b) $0.746 < 0.8$

c) $2.01 = 2.01$

d) $15.7 > 15.64854$

6) Ordering decimal numbers

1) Arrange the following numbers in descending order.

$$2.1 , 2.014 , 2.0 , 2.54$$

$\Longrightarrow$ **2.54 , 2.1 , 2.014 , 2.0**

2) Re-arrange the numbers below in ascending order.

$$3.7 , 3.08 , 3.2648 , 3.4$$

$\Longrightarrow$ **3.08 , 3.2648 , 3.4 , 3.7**

7) Expressing decimals as fractions

Express the following decimals as fractions in their lowest terms:

a) $0.45 = \dfrac{45}{100} = \dfrac{9}{20}$

b) $1.5 = \dfrac{15}{10} = \dfrac{3}{2} = 1\dfrac{1}{2}$

c) $0.04 = \dfrac{4}{100} = \dfrac{1}{25}$

d) $0.45 = \dfrac{45}{100} = \dfrac{9}{20}$

e) $2.25 = \dfrac{225}{100} = \dfrac{9}{4} = 2\dfrac{1}{2}$

EXERCISES

Express the following decimals into fractions and simplify:

a) 0.5

b) 0.75

c) 0.625

d) 20.4

e) 3.75

h) 1.45

f) 0.025

i) 0.125

g) 1.8

j) 1.25

8) **Rounding off decimal numbers**

1) Round off 0.45 to the nearest tenths.

0.4̲5

_+1__

0.50 = 0.5

2) Correct 2.39246 to the nearest whole number.

2̲.39246

+0_____

2.00000 = 2

3) Correct 0.2485 to two decimal places.

0.2485

__+1__

0.2500 = 0.25

Exercises

1) Correct 34.728 to one decimal place.
2) Round off 0.493 to the nearest hundredths.
3) Correct 5.962 to the nearest tenths.
4) Round off 62.4896 to the nearest whole number.
5) Correct 0.2374 to two decimal places.

9) **Operations in decimals**

a) **Addition**

When adding decimals, arrange them vertically ensuring that the points are aligned.

Examples

Add: 1) 8.7501 + 21.2 =

8.7501
+21.2_____
29.9501

2) 23.96 + 6.4

23.96
+6.4
30.36

3) 6 + 2.45

6.00
+2.45
8.45

Examples

Add:

a) 6.3 + 4.5

b) 10.6 + 5.31

c) 3.5 + 17.285

d) 3.83 + 28

e) 7.1 + 8.21

f) 3.703 + 5.6

g) 10.5 + 0.9

h) 15.1 + 7.5

i) 8 + 14.6

j) 56.86 + 2.2

b) Subtraction

Arrange the numbers ensuring that the decimal points are aligned.

Examples

Subtract: 1) 46.4 − 8.2465 =

46.4000
−8.2465
38.1535

2) 17 + 3.72

17.00
−3.72
13.28

Exercises

Subtract:

a) 10.5 − 0.9

b) 15.4 − 7.42

c) 45 − 9.4

d) 41.5 − 7.52

e) 5.6 − 3.702

f) 65.2853 − 8.95

g) 0.7 − 0.005

h) 7 − 3.62

More about addition and subtraction

Work out:

a) 3.5 − 7.4 + 6.2 = **(3.5 + 6.2) − 7.4**

         = **9.7 − 7.4**

         = **2.3**

b) 4 − 6.25 + 5 = **(4 + 5) − 6.25**

$$= 9 - 6.25$$
$$= 2.75$$

c) $2.56 - 8.486 + 12.4 = (2.56 + 12.4) - 8.486$
$$= 14.96 - 8.486$$
$$= 6.474$$

Exercises

Work out:

a) $4 - 5.72 + 3$

b) $7.3 - 10.68 + 5$

c) $5.8 - 8 + 11.93$

d) $8 - 9.12 + 3.25$

e) $7.12 - 15 + 13.4$

3) Multiplication

Multiply in ordinary, but the product must have the number of decimal places equal to those in the multiplicand and multiplier.

Examples

Multiply:

a) 2.35×6.4

$$\begin{array}{r} 2.35 \\ \times 6.4 \\ \hline 1410 \\ +940 \\ \hline 15.04 \end{array}$$

b) 26.856×8

$$\begin{array}{r} 26.856 \\ \times 8 \\ \hline 214.848 \end{array}$$

Examples

Work out the following:

a) 6.5×1.2

b) 7.5×0.16

c) 18.65×0.32

d) 7.25×44

e) 0.234×0.15

f) 0.008×6.5

g) 27×4.8

h) 3.75×0.4

Quick multiplication

1) Quick multiplication by 10, 100, 1000, 10,000 etc

Multiply:

a) $9.264 \times 100 =$ **926.4**

b) $2.4 \times 10 =$ **24**

c) $7.5 \times 100 = \mathbf{750}$

d) $0.4 \times 1000 = \mathbf{400}$

e) $3.027 \times 10{,}000 = \mathbf{30\ 270}$

2) Quick multiplication by 0.25, 0.5 and 0.75

Express **- 0.25 as** $\frac{1}{4}$

- 0.5 as $\frac{1}{2}$

- 0.75 as $\frac{3}{4}$

Examples

Work out:

a) $444 \times 0.25 = \mathbf{444} \times \frac{1}{4} = \mathbf{111}$

b) $840 \times 0.5 = \mathbf{840} \times \frac{1}{2} = \mathbf{420}$

c) $280 \times 0.75 = \mathbf{280} \times \frac{3}{4} = \mathbf{210}$

4) Division

When dividing a decimal by a decimal, make the **divisor** a whole number.

Exercises

Work out:

a) $0.08 \div 4 = \mathbf{0.02}$ or $\mathbf{0.08 \div 4} = \frac{8}{100} \div \frac{4}{1}$

$$= \frac{8}{100} \times \frac{1}{4}$$

$$= \frac{2}{100}$$

$$= \mathbf{0.02}$$

```
        0.02
      _____
  4 ) 0.08
       0
       00
       0
       08
       8
       0
```

b) $20.48 \div 0.2 = \mathbf{204.8 \div 2} =$ or $20.48 \div 0.2 = \frac{2048}{100} \div \frac{2}{10}$

$$= \frac{2048}{100} \times \frac{10}{2}$$

$$= \frac{10240}{100}$$

```
        102.4
      _____
  2 ) 204.8
```

$$\underline{2}$$
$$00$$
$$\underline{0}$$
$$04$$
$$\underline{4}$$
$$08$$
$$\underline{8}$$
$$0$$

= 102.4

Exercises

Work out the following:

a) 1.2 ÷ 0.6 =

b) 1.2 ÷ 0.6 =

c) 8.1 ÷ 0.027 =

d) 3.9 ÷ 0.03 =

e) 48.8 ÷ 4 =

f) 3636 ÷ 0.6 =

g) 0.048 ÷ 0.12 =

h) 0.204 ÷ 0.6 =

i) 59.5 ÷ 0.07 =

j) 1.2 ÷ 0.6 =

k) 0.4 ÷ 0.002 =

l) 1.2 ÷ 0.6 =

Quick division by 10, 100, 1000 etc

Calculate:

a) 25.6 ÷ 10 = **2.56**

b) 784 ÷ 100 = **7.84**

c) 4000 ÷ 1000 = **4**

d) 45 ÷ 100 = **0.45**

e) 82.6 ÷ 10000 = **0.00826**

Quick division by 0.25, 0.5 and 0.75

Express - **0.25 as** $\frac{1}{4}$

- **0.5 as** $\frac{1}{2}$

- **0.75 as** $\frac{3}{4}$

Examples

Work out: **a)** $45 ÷ 0.25 = \mathbf{45} ÷ \frac{1}{4} = \mathbf{45} \times \frac{4}{1} = \mathbf{180}$

b) $750 ÷ 0.5 = \mathbf{750} ÷ \frac{1}{2} = \mathbf{750} \times \frac{2}{1} = \mathbf{1500}$

c) $66 ÷ 0.75 = \mathbf{66} ÷ \frac{3}{4} = \mathbf{66} \times \frac{4}{3} = \mathbf{88}$

More about operation in decimal numbers

Work out: $\frac{0.4 \times 0.03}{0.006}$ rom 7;21-23

Numerator $= \mathbf{0.4} \times \mathbf{0.03} = \dfrac{4}{10} \times \dfrac{3}{100} = \dfrac{3}{250}$

Denominator = 0.006 $= \dfrac{6}{1000} = \dfrac{3}{500}$

Whole statement = numerator ÷ denominator

$$= \dfrac{3}{250} \div \dfrac{3}{500} = \dfrac{3}{250} \times \dfrac{500}{3} = \mathbf{2}$$

Exercises

Work out:

1) $\dfrac{0.2 \times 0.04}{0.012}$

2) $\dfrac{0.04 \times 0.2}{0.004}$

3) $\dfrac{0.24 \times 0.6}{0.16}$

4) $\dfrac{0.72 \times 0.96}{0.0144}$

5) $\dfrac{0.25 \times 3.6}{0.45}$

6) $\dfrac{0.7 \times 0.6}{0.3}$

7) $\dfrac{4.5 \times 1.6}{1.5}$

8) $\dfrac{2.4 \times 0.54}{0.36}$

9) $\dfrac{0.04 \div 0.2}{0.05}$

10) $\dfrac{27.515 + 2.485}{0.5}$

11) $\dfrac{0.4 \div 0.05}{0.2}$

12) $\dfrac{0.08 \div 0.4}{0.05}$

13) $\dfrac{0.75 \times 0.3}{0.025}$

14) $\dfrac{0.1 \times 0.36}{0.09}$

15) $\dfrac{0.3 \times 0.8}{0.12}$

16) $\dfrac{0.09 \times 0.4}{0.06}$

17) $\dfrac{0.24 \times 0.3}{0.036}$

18) $\dfrac{0.9 \div 0.75}{0.08}$

Word problems involving decimals

1) Ogolla ate 0.5 of his apple in the morning and 0.2 of it in the afternoon. How much of his apple did he eat altogether?

2) Opion bought 4 metres of cloth and used 2.3 metres for making shirts. Find the length of the remaining cloth.

3) Mandela weighs 94.2 kg and his brother weighs 85.2 kg. What is the difference between their weights?

4) A piece of cloth 16 m long is cut into small pieces of 0.4 m. how many pieces will a tailor make?

5) A car was filled with 18 litres of petrol. It it uses 0.9 litres each kilometer, how many kilometres will it cover?

SQUARE ROOT OF DECIMALS

When finding the square root of a decimal, first express it as a fraction.

Examples

1) Find the square root of 0.81.

$$\sqrt{0.81} = \sqrt{\frac{81}{100}} = \frac{\sqrt{81}}{\sqrt{100}} = \frac{9}{10} = 0.9$$

2) Calculate: $\sqrt{2.25} + \sqrt{1.44} =$

$$\sqrt{2.25} + \sqrt{1.44} = \sqrt{\frac{225}{100}} + \sqrt{\frac{144}{100}} = \frac{\sqrt{225}}{\sqrt{100}} + \frac{\sqrt{144}}{\sqrt{100}} = \frac{15}{10} + \frac{12}{10} = 1.5 + 1.2 = 2.7$$

3) Work out: $\dfrac{\sqrt{3.24} - \sqrt{2.56}}{\sqrt{0.64}} =$

4) Evaluate: $\dfrac{\sqrt{0.49} + \sqrt{0.25}}{\sqrt{0.16}} =$

5) Simplify: $\sqrt{0.36} + \sqrt{1.69}$

TOPIC 14: <u>SPEED, DISTANCE AND TIME</u>

1) <u>Comparing 12-hour format to the 24-hour format</u>

12 hour clock system	24 hour clock system	
		Midnight
12:00 am	00:00 hrs	
1:00 am	01:00 hrs	
2:00 am	02:00 hrs	
3:00 am	03:00 hrs	
4:00 am	04:00 hrs	
5:00 am	05:00 hrs	
6:00 am	06:00 hrs	
7:00 am	07:00 hrs	Midday
8:00 am	08:00 hrs	
9:00 am	09:00 hrs	

10:00 am	10:00 hrs
11:00 am	11:00 hrs
12:00 am	12:00 hrs
1:00 pm	13:00 hrs
2:00 pm	14:00 hrs
3:00 pm	15:00 hrs
4:00 pm	16:00 hrs
5:00 pm	17:00 hrs
6:00 pm	18:00 hrs
7:00 pm	19:00 hrs
8:00 pm	20:00 hrs
9:00 pm	21:00 hrs
10:00 pm	22:00 hrs
11:00 pm	23:00 hrs
12:00 am	00:00 hrs

2) <u>Converting 12-hr to 24-hr format</u>

1) Convert 6:24 a.m to the 24-hr format.

$$\begin{array}{r} 6{:}24\ am \\ +\ 00{:}00 \\ \hline 06{:}24\ hrs \end{array}$$

2) Express 11:28 pm as 24 hour system.

$$\begin{array}{r} 11{:}28\ pm \\ +\ 12{:}00 \\ \hline 23{:}28\ hrs \end{array}$$

3) Convert 4:02 a.m to the 24-hr system.

4) Change 3:30 p.m to the 24-hr format.

5) Change the time from the 12-hour format to the 24-hour format.

a) 4:21 p.m   c) 1:59 p.m   e) 5:56 a.m

b) 8:45 a.m   d) 2:43 p.m   f) 5:00 p.m

g) 7:18 a.m   i) 9:12 p.m   k) 8:00 p.m

h) 12:46 a.m   j) 10:43 a.m   l) 7:34 a.m

3) <u>Converting 24-hr to 12-hr format</u>

1) Change 06:46 hr to the 12-hour format.

$$06:46 \; hrs$$
$$- \; 00:00$$
$$6:46 \; am$$

2) Convert 20:25 hr to the 12-hour system.

$$20:25 \; hrs$$
$$- \; 12:00$$
$$8:25 \; am$$

3) Express 07:06 hr as 12-hour clock system.

4) Convert 14:37 hr to the 12-hour format.

5) Convert the following times to 12 hour clock system.

a) 10:00 hr	f) 23:57 hr	k) 02:20 hr
b) 04:10 hr	g) 09:30 hr	l) 13:40 hr
c) 17:38 hr	h) 15:54 hr	m) 09:17 hr
d) 19:40 hr	i) 20:04 hr	n) 22:00 hr
e) 01:54 hr	j) 05:07 hr	o) 08:50 hr

4) <u>The concept of time zones</u>

- **Time zone =** The range of longitude where the standard time is used.
- The world is divided into **24 time zones**. Each time zone is 15^0 apart.
- There is a difference of 1 hour between each time zone, **15^0 = 1 hour i.e 1^0 = 4min**
- All countries in the same time zone keep the same time.
- Every 15^0 due East you gain 1 hour, and 15^0 due West you lose 1 hour.
- We can calculate the time of any place in the world if its time zone or line longitude is known.

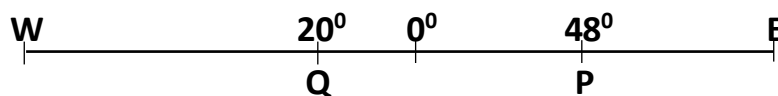### <u>Exercises</u>

1) It is 11:30 p.m in Cairo. What time is it in New York which is located in the 7th time zone to the west of Rwanda?

   7th time zone = 7 hrs

   At 11:30 pm – hrs = 4:30 pm
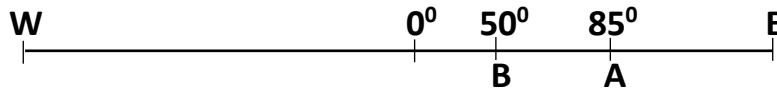
2) It is 11:30 pm at town Q 20^0 W. What I the time at P 48^0 E?

W ────────── 20^0 ── 0^0 ────── 48^0 ────── E

   Q   P

   Degrees =20^0 + 48^0 = 68^0

Time = $68^0 \times 4$ min = 272 min = 4 hrs 32min

At 11:30 pm + 4hrs 32min = 16:02 am − 12:00 = 4:02 am

3) It is 4:00 a.m in town A 85^0 E. What time is it at town B 30^0 E?

W 0^0 50^0 85^0 E

                   B   A

Degrees = 85^0 - 50^0 = 35^0

Time = $35^0 \times 4$ min = 140 min = 2 hrs 20min

At 4:00 am - 2hrs 32min = 1:28 am

4) It is 8:30 pm at place Q 20^0 W. What time is it at at P 48^0 E?

5) The time in Sydney is 10:00 a.m. Baghdad is in the 6th time zone to the West of Sydney. What is the time in Bagdad?

6) Karegeya kives at a place 78^0 E and his time is 8:20 pm. His friend Umukundwa lives at longitude 15^0 E. What is Umukundwa's corresponding time?

7) It is 8:20 am in Town B. what time is Town A which is a longitude 60^0 E from B?

8) The time in Accra is midnight. What time is it in Buenos Aires, in the 4th time zone to the West?

9) It is 10:40 pm at town R 30^0 W. What time is at town N 45^0E?

10) It is 7:20 am at town N 63^0 E. What time is at town N 20^0E?

5) <u>Finding duration</u>

1) How long is it from 10:00 a.m to 2:00p.m?

10:00am   **11:00am**   **12:00pm**   **1:00pm**   **2:00pm**

       1h   **2h**   **3h**   **4h**

Duration = 4 hrs
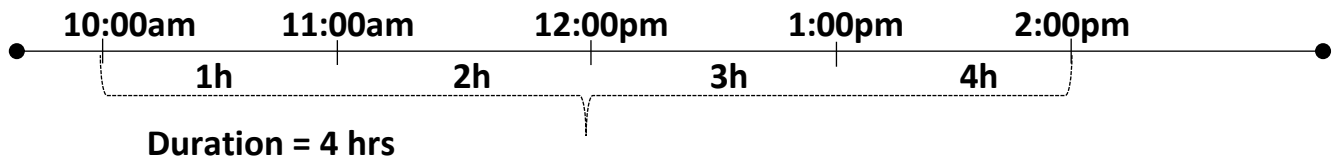
2) A doctor was on duty in hospital from 9:00 a.m to 12:00 noon. How long was she on for duty?

3) It started raining at 8:40 a.m and stopped at 11:30 a.m. For how long did it rain?

8:40am   **9:40am**   **10:40pm**

       1h   **2h**   **+ 50 min**

Duration = 2 hrs 50 min
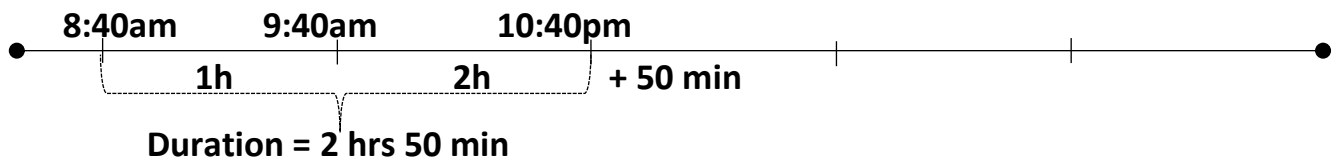
<u>Exercises</u>

1) A baby slept at 8:15 p.m and woke up at 10:15 p.m. How long did the baby sleep?

2) A train left from one station at 10:45 am. It arrived at its destination at 12:45 pm. Calculate the time it took.

3) A class had lessons from 7:30 a.m to 3:30 pm. How long did the lessons take?

4) A football match started at 2:00 p.m and ended at 3:30 p.m. How long did the match take?

5) A car left Kigali at 3:20 a.m and reached its destination at 7:50 a.m. How long did it take?

6) How many hours are there from

a) 9:00 a.m to 3:00 p.m?   c) 4:30 p.m to 8:10 p.m?   e) 8:20 a.m to 11:10 a.m

b) 6:00 a.m to 4:00 p.m?   d) 10:00 p.m to 2:00 a.m   f) 9:00 a.m to 3:00 a.m

6) Converting units of time

Division of time

- ❖ 1 minute = 60 seconds
- ❖ 1 hour = 60 minutes
- ❖ 1 week = 24 hours
- ❖ 1 fortnight = 2 weeks
- ❖ 1 month = 4 weeks = 30 days
- ❖ 1 year = 12 months = 360 days
- ❖ 1 decade = 10 years
- ❖ 1 century = 100 years
- ❖ 1 millennium = 1000 years

a) Converting days into hours

1) How many hours are there in 6 days?

1 day = 24 hrs

6 days = 6 × 24 hrs = 144 hrs

2) Change 15 days into hours.

1 day = 24 hrs

15 days = 15 × 24 hrs = 360 hrs

b) Converting hours into days

1) How many days are there in 192 hours?

1 day = 24 hrs

192 hrs = 192 ÷ 24 = 8 days

2) Convert 176 hours into days.

1 day = 24 hrs

176 hrs = 176 ÷ 24 = 7 days 8 hrs

Exercises

1) A boarding school went for mid-term holidays for 144 hours. How many days was the holiday?

2) A youth camp took place over 312 hours. How many days did the camp take?

3) An activity of making a road took one week in a certain place. How many hours did it take?

4) Convert the following hours into days

a) 74 hours	d) 120 hours	g) 72 hours
b) 168 hours	e) 231 hours	h) 83 hours
c) 119 hours	f) 240 hours	i) 217 hours

5) Change the following days into hours

a) 5 days	d) 30 days	g) 2 days
b) 10 days	e) 16 days	h) 8 days
c) 18 days	f) 3 days	i) 38 days

c) Converting hours into minutes

1) A tourist took 6 hours in her visit to the animal park. How many minutes did the tourist take in the park?

1 hr = 60 min

6 hrs = 6 × 60 min = 360 min

2) How many minutes are there in 38 hours?

38 hrs = 38 × 60 min = 2,280 min

d) Converting minutes into hours

1) How many hours are there in 480 minutes?

1 hr = 60 min

480 min = 480 ÷60 = 8 hrs

2) Complete: 140 min = …………..hrs………..min

140 min ÷60 = 2 hrs remainder 20 min

140 min = 2 hrs 20min

3) A national drama festival lasted 200 minutes. How many hours and minutes did it last?

200 min = 200 ÷60 = 3 hrs 20 min

Exercises

1) A traditional music festival lasted for $3\frac{1}{2}$ hours. How many minutes did the festival last?

2) A football match took 100 minutes. How many hours and minutes did it take?

3) Convert the following into minutes
   a) 5 hrs
   b) 12 hrs
   c) $4\frac{1}{5}$ hrs
   d) 37 hrs
   e) $7\frac{1}{3}$ hrs
   f) 80 hrs
   g) 24 hrs
   h) 10 hrs
   i) $5\frac{2}{3}$ hrs

4) Complete the following:
   a) 547 min =……….hrs………min
   b) 193 min = ………..hrs ……..min
   c) 318 min = …………hrs……….min

5) Change into hours:
   a) 540 min
   b) 420 min
   c) 840 min
   d) 720 min
   e) 240 min
   f) 2340 min
   g) 900 min
   h) 960 min
   i) 300 min

e) Converting minutes into seconds

1) How many seconds are in 4 minutes?

   1 min = 60 sec

   4 min = 4 × 60 = 240 sec

2) English examination took started at 4:00 am and ended at 11:00 am. How many seconds did it take?

   Duration = 11:00 am − 4:00 am = 7 hrs

   7 hrs = 7 × 60 = 420 sec

f) Converting seconds into minutes

1) How many minutes are in 1200 seconds?

   1 min = 60 sec

   1200 sec = 1200 ÷ 60 = 20 min

2) Complete: 315 sec = ……….min………sec

   315 sec ÷ 60 = 5 min remainder 15 sec

   315 sec = 5 min 15 sec

Exercises

1) How many minutes are there in 600 seconds?

2) How many seconds are there in 14 minutes?

3) Complete:

a) 635 sec = ………min…….sec

b) 149 sec = ………min……..sec

c) 803 sec = ………min……….sec

d) 274 sec = ………min………..sec

g) Converting hours into seconds

1) Convert 3 hours into seconds.

1 hr = 60 min = 60 × 60 sec = 3600 sec

Therefore, conversion fact: 1h = 3600 sec

3 hrs = 3 × 360 0 sec = 10,800 sec

2) A church prayer session lasted for $5\frac{1}{2}$ hours. How many seconds did it last?

$5\frac{1}{2}$ hrs = $\frac{11}{2}$ × 360 0 sec = 19,800 sec

Exercises

1) A bus took 10 hrs 30 sec to travel from town A to town B. Find the time in seconds that the bus took.

2) A tractor took 2 hours to dig a piece of land. Calculate how much time a tractor took in seconds.

3) Convert the following into seconds:

a) 5 hours

b) 12 hours

c) $4\frac{1}{3}$ hours

d) 34 hours

e) $8\frac{1}{4}$ hours

f) 7 hours

h) Converting days, hours and minutes into seconds

Complete:

1) 12 hrs 4 min = **43,440 sec**

$$\begin{array}{ccc} 12\ hrs & 4\ min & 43\ 200\ sec \\ \times\ 60 & \times\ 60 & +\ \ 240\ sec \\ \hline 720\ min & 240\ sec & 43\ 440\ sec \\ \times\ 60 & & \\ \hline 43200 sec & & \end{array}$$

2) 6 days 5 hrs 24 min 20 sec =

$$
\begin{array}{r}
6\ days \\
\times\ 24 \\
\hline
144\ hrs \\
\times\ 60 \\
\hline
8640\ min \\
\times\ 60 \\
\hline
518\ 400\ sec
\end{array}
\qquad
\begin{array}{r}
5\ hrs \\
\times\ 60 \\
\hline
300\ min \\
\times\ 60 \\
\hline
1800\ sec
\end{array}
\qquad
\begin{array}{r}
20\ min \\
\times\ 60 \\
\hline
1200 sec
\end{array}
\qquad
\begin{array}{r}
518\ 400 \\
+18000 \\
1200 \\
\hline
537\ 860 sec
\end{array}
$$

Exercises

Complete:

a) 4 hrs 50 min = ………………sec

b) 2 hrs 10 min = ………………sec

c) 7 hrs 30 min = ………………sec

d) 3 hrs 15 min 10 sec = …….sec

e) $2\frac{1}{2}$hrs 40 min = ………………sec

f) 5 hrs 45 min = ………………sec

i) Converting seconds to days, hours and minutes

Complete the following:

642 575 sec = **7** days **10** hrs **29** min **35** sec

```
   10709 min              178 hrs               7 days
60)642575            60 ) 10709           24 ) 178 hrs
  −60                   - 60                  - 168
   042                   470                  10 hrs
  − 0                   - 420
   425                   509
  −420                  - 480
    57                   29 min
   − 0
   575
  −540
   35 sec
```

Exercises

Complete the following:

a) 97 235 sec = __days__hrs__min__sec

b) 48 217 sec = ___hrs___min___sec

c) 105 539 sec = __days__hrs__min__sec

d) 9 428 sec = ___hrs___min___sec

e) 87 351 sec = __days__hrs__min__sec

f) 16 302 sec = ___hrs___min___sec

g) 98 612 sec = __days__hrs__min__sec

h) 15 524 sec = ___hrs___min___sec

7) Operations in time measurements

a) Addition

Work out the following:

a) 4 hrs 40 min 30 sec + 5 hrs 30 min 50 sec = **10 hrs 11 min 20 sec**

1   1

$$\begin{array}{r} \text{4 hrs 40 min 30 sec} \\ + \text{5 hrs 30 min 50 sec} \\ \hline \text{10 hrs 11 min 20 sec} \end{array}$$

b) 45 min 28 sec + 34 min 36 sec =

$$\begin{array}{r} \text{1} \quad \text{1} \qquad \text{1} \\ \text{45 min 28 sec} \\ + \text{ 34 min 36 sec} \\ \hline \text{1hr 20 min 4 sec} \end{array}$$

Exercises

1) Add the following:

   a) 7 h 55 min + 2 h 15 min =

   b) 5 h 45 min + 1 h 27 min =

   c) 16 h 38 min 52 sec + 9 h 42 min 18 sec =

   d) 14 h 18 min + 12 h 42 min =

2) A mathematics lesson started at 8.40 a.m. It lasted for 1h 20 min. At what time did the lesson end?

3) We went for lunch at 12.45 p.m. We took a 1 h 15 min lunch break. When did the lunch break end?

b) Subtraction

Subtract the following:

   a) 8 h 15 min 30 sec − 5 h 40 min 45 sec = **2hr 34 min 45 sec**

$$\begin{array}{r} \text{7} \quad \text{14} \quad \text{90} \\ \cancel{8} \text{ h } \cancel{15} \text{ min 30 sec} \\ - \text{5 h 40 min 45 sec} \\ \hline \text{2hr 34 min 45 sec} \end{array}$$

   b) 5 hrs − 2 hrs 40 min =

$$\begin{array}{r} \text{4} \\ \cancel{5} \text{ h 60 min} \\ - \text{2 h 40 min} \\ \hline \text{2h 20 min} \end{array}$$

Exercises

1) Work out:

   a) 4 h − 1 h 30 min =

   b) 18 h 35 min − 11 h 45 min =

b) A cross-country race ended at 12.35 p.m. The duration of the race was 2h 10min. At what time did the race begin?

c) Multiplication

Work out:

   1) 2 hrs 35 min 40 sec × 4 =

$$\frac{\begin{array}{r} \textbf{2 hrs 35 min 40 sec} \\ \times \textbf{4} \end{array}}{\textbf{10 hrs 22 min 40 sec}}$$

2) 8 h 30 min 20 sec × 5 =

$$\frac{\begin{array}{r} \textbf{2 hrs 35 min 40 sec} \\ \times \textbf{5} \end{array}}{\textbf{1 dy 18 hrs 31 min 40 sec}}$$

Exercises

Multiply: a) 3 hrs 15 min 20 sec × 3 =

b) 6 hrs 25 min 50 sec × 2 =

Speed

❖ A speed is a rate of movement from one place to another.

❖ Speed is got by dividing the distance moved by the time taken.

❖ A speed is expressed either in **km/h** or **m/s**.

Examples

1) A motor cyclist travelled for 3 hours and covered a distance of 210 kilometres. What speed was he moving?

D = 210 km

T = 3 hrs

$$\textbf{S} = \frac{D}{S} = \frac{\textbf{210 km}}{\textbf{3 hrs}} = \textbf{70 km/h}$$

2) An athlete ran 600 m in 5 minutes. Calculate his speed in km/h.

D = 600 m

T = 5 min

$$\textbf{S} = \frac{D}{S} = \frac{\textbf{600 m}}{\textbf{5 min}} = \frac{\textbf{600 m}}{\textbf{300 sec}} = \textbf{2 m/sec}$$

3) A bus covered 40 km in 20 minutes. Find the speed.

D = 40 km

T = 20 min

$$\textbf{S} = \frac{\textbf{40 km}}{\textbf{20 min}} = \frac{\textbf{40 km}}{\frac{\textbf{20 hr}}{\textbf{60}}} = \textbf{40 km} \div \frac{\textbf{20 hr}}{\textbf{60}} = \textbf{40 km} \times \frac{\textbf{60}}{\textbf{20 hr}} = \textbf{120 km/h}$$

4) A taxi travelled 80 km in 2hr 30min. what was the speed.

D = 80 km

T = 2 hr 30 min = 150 min

$$\textbf{S} = \frac{\textbf{80 km}}{\textbf{150 min}} = \frac{\textbf{80 km}}{\frac{\textbf{150 hr}}{\textbf{60}}} = \textbf{80 km} \div \frac{\textbf{150 hr}}{\textbf{60}} = \textbf{80 km} \times \frac{\textbf{60}}{\textbf{150 hr}} = \textbf{32 km/h}$$

Exercises

1) A car travelled 100 km in $1\frac{1}{4}$ hours. Find its speed.

2) Musa moved 90 km in 30 min. find his speed.

3) Pauline left Kigali for Musanze at 9:00 am and reached Musanze at 10:30 am if Musanze is 87 km far from Kigali, find the speed.

4) Sarah started the journey at 7:20 am and reached her destination at 9:50 am. Find the speed if the journey was 225 km.

5) At what distance was the car travelling if it covered 540 km 6 hours?

6) A driver covers 90 km in 50 minutes. At what speed was he moving?

7) Mutima took $2\frac{1}{2}$ h to complete a distance of 450 km. find the speed.

8) A car travelled 80 km in 1 hour 30 minutes. Find the speed

Converting speed from km/h to m/s

1) Express 72 km/h as m/sec.

Change km to m and hr to sec

$$72 \text{ km/h} = \frac{72 \text{ km}}{h} = \frac{72\,000 \text{ m}}{3600 \text{ sec}} = 20 \text{ m/sec}$$

2) Convert 90 km/h into m/sec.

$$90 \text{ km/h} = \frac{90 \text{ km}}{h} = \frac{90\,000 \text{ m}}{3600 \text{ sec}} = 25 \text{ m/sec}$$

Exercises

1) Express 108 km/hr as m/sec.

2) Change 54 km/hr into m/sec.

3) Change the following speeds to m/sec.

a) 18k/hr	e) 252 km/hr	i) 27 km/hr
b) 36km/hr	f) 60 km/h	j) 63 km/hr
c) 144 km/hr	g) 9 km/hr	k) 81 km/h
d) 180 km/hr	h) 216 km/hr	l) 117 km/hr

4) The distance from village A to village B is 720 km. a car takes 6 hours to cover the journey. Calculate its speed in m/sec.

5) A car travelled 50 km in 2 hours. Find its speed in m/sec.

Converting speed from m/sec to km/hr

1) Express 200 m/sec as km/hr.

Change m to km and sec to hr.

$$200 \text{ m/sec} = \frac{200 \text{ m}}{sec} = \frac{\frac{200}{1000} km}{\frac{1}{3600} hr} = \frac{200 km}{1000} \div \frac{1}{3600} = \frac{200 km}{1000} \times \frac{3600}{1} = 720 \text{ km/hr}$$

2) Change 10 m/sec into km/hr.

$$10 \text{ m/sec} = \frac{10 \text{ m}}{sec} = \frac{\frac{10}{1000}km}{\frac{1}{3600}hr} = \frac{10km}{1000} \div \frac{1}{3600} = \frac{10km}{1000} \times \frac{3600}{1} = 36 \text{ } km/hr$$

Exercises

1) Express 45 m/sec as km/hr.

2) Convert the following into km/hr.

a) 15 m/sec	d) 40 m/sec	g) 30 m/sec
b) 50 m/sec	e) 25 m/sec	h) 5 m/sec
c) 45 m/sec	f) 100 m/sec	i) 20 m/sec

3) In a school competition, one athlete ran 100 metres in 15 seconds. What was the speed in km/hr?

6) Distance

A distance is the amount of space between two places.

Examples

1) Calculate the distance travelled by a bus in 4 hours at 80 km/hr.

   T = 4 hrs

   S = 80 km/hr

   D = S × T = 80 km/hr × 4 hrs = 320 km

2) Find the distance covered by a car moving at 40 km/h in 30 minutes.

   S = 40 km/hr

   T = 30 min

   D = S × T = 40 km/hr × 30 min = $\frac{40 \text{ } km}{hr}$ × 30 min = $\frac{40 \text{ } km}{60 \text{ } min}$ × 30 min = 80 km

3) A bus took 1 hour 20 minutes to travel from town A to town B travelling at 45 km/h. Find the distance between town A and town B.

   S = 45 km/hr

   T = 1 hr 20 min = 80 min

   D = S × T = 45 km/hr × 80 min = $\frac{45 \text{ } km}{hr}$ × 80 min = $\frac{45 \text{ } km}{60 \text{ } min}$ × 80 min = 60 km

4) A car travelled 150 km in 3 hours. What distance can it travel in 4 hours?

   3 hrs ⟶ $\frac{150 \text{ } km \times 4}{3} = 200 \text{ } km$

   4 hrs

5) A bus took 1 hour 20 minutes to travel from town A to town B travelling at 45 km/h. Find the distance between town A and town B.

   S = 45 km/hr

   T = 1 hr 20 min = 80 min

   D = S × T = 45 km/hr × 80 min = $\frac{45 \text{ } km}{hr}$ × 80 min = $\frac{45 \text{ } km}{60 \text{ } min}$ × 80 min = 60 km

Exercises

1) A motorist started his journey at 8:20 a.m and stopped at 1:30 p.m. He travelled at an average speed of 17 km/h. How long was the journey?
2) Kazungu drove at a speed of 36 km/h for 4 hours. What distance did he cover?
3) Find the distance covered by a bus moving at 60 km/h in 1 hour 30 minutes.
4) A motorcyclist drove at a speed of of 70 km/h. What distance did he cover in $2\frac{1}{2}$ hours?
5) Mugenzi runs 60 km in 3 hours. What distance can he cover in 30 minutes?
6) Nkubito took 15 minutes to walk from home to school while walking at 40 metres per minute. How far is the school from his home?

7) Time

Time means how long somebody or something takes to complete a certain a distance.

Exercises

1) How long does it take a car to cover a distance 180 km at 45 km/hr?

   D = 180 km

   S = 45 km/hr

   T = $\frac{D}{S} = \frac{180\ km}{45\ km/hr} = 4\ hrs$

2) A bus travelling at a speed of 80 km/hr covered a distance of 60 km. Find the time.

   D = 60 km

   S = 80 km/hr

   T = $\frac{D}{S} = \frac{60\ km}{80\frac{km}{hr}} = \frac{3}{4}\ hrs \quad or \ \frac{3}{4} \times 60\ min = 45\ min$

Exercises

1) After how much longer can a distance of 480 km be covered at 40 km/hr than at 60 km/hr?
2) At 40 km/h, a lorry covered a distance of 320 km. In how many hours can the same distance be covered at 60 km/h?
3) From Kibuye to Kigali a bus moved at a speed of 50 km/h over a distance of 200 km. how long did it take the bus to cover the journey?
4) A car travelling at a speed of 100 km/h covered a distance of 150 km. calculate the time.
5) What time did Mutoni use to travel a distance of 120 km with a speed of 20 m/sec?

8) <u>Average speed</u>

1) A car covers 120 km in 3 hours and then another 60 km in 2 hours. Calculate the average speed for the whole journey.

D_1 = 120 km

T_1 = 3 hrs

D_2 = 60 km

T_2 = 2 hrs

Total D = D_1 + D_2 = 120 km + 60 km = 180 km

Total T = T_1 + T_2 = 3 hrs + 2 hrs = 5 hrs

Average speed = $\frac{Total\ D}{Total\ T} = \frac{180\ km}{5\ hrs} = 36\ km/hr$

2) A lorry covers a distance in 4 hours travelling at 40 km/hr and later covers 120 km moving at 20 km/h. Find its average speed.

T_1 = 4 hrs

S_1 = 40 km/h

D_1 = S × T = 40 km/h × 4 hrs = 160 km

D_2 = 120 km

S_2 = 20 km/h

$T_2 = \frac{D}{S} = \frac{120\ km}{20\ km/h} = 6\ hrs$

Total D = D_1 + D_2 = 160 km + 120 km = 280 km

Total T = T_1 + T_2 = 4 hrs + 6 hrs = 10 hrs

Average speed = $\frac{Total\ D}{Total\ T} = \frac{280\ km}{10\ hrs} = 28\ km/hr$

3) A taxi took 6 hours to travel from town Q to town P moving at 50 km/hr. On the returning journey the taxi took 4 hours. Calculate the average speed for the whole journey.

T_1 = 6 hrs

S_1 = 50 km/h

D_1 = S × T = 50 km/h × 6 hrs = 300 km

D_2 = D_1 = 300 km

T_2 = 4 hrs

Total D = D_1 + D_2 = 300 km + 300 km = 600 km

Total T = T_1 + T_2 = 6 hrs + 4 hrs = 10 hrs

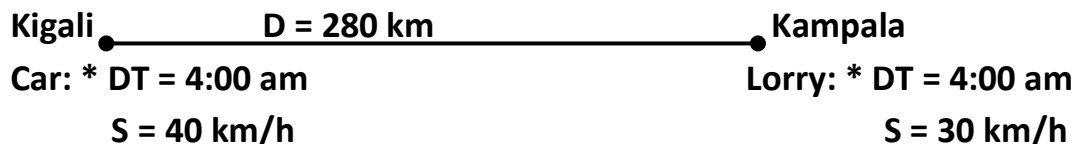Average speed = $\frac{Total\ D}{Total\ T} = \frac{600\ km}{10\ hrs} = 60\ km/hr$

Exercises

1) A bus takes 6 hours to cover a distance moving at 80 km/h. it returns moving at 120 km/h. What is the average speed?

2) A car takes 2 hours to travel from Gatuna to Kigali a distance of 78 km and 4 hours on the return journey to Gatuna. Calculate the average speed of the whole journey.

3) Two towns Q and P are 180 km apart. A car takes 3 hours to move from town Q to town P and 2 hours to return. What is the average speed of the whole journey?

4) Kayitesi took 3 hrs to travel from town A to B at 80 km/h. She then continued to town C at a speed of 48 km/h for 2 hrs. Find the average speed.

5) A car took 4 hours to cover a journey at a speed of 45 km/h and another 5 hours to return through the same distance. Find the average speed for the whole journey.

6) It took a truck $3\frac{1}{2}$ hours to cover a distance travelling at 40 km/h. It returned moving at 56 km/h. Calculate its average speed for the whole journey.

8) Moving bodies towards each other

A car left Kigali for Kampala at 4:00 am travelling at 40 km/h. At the same time a Lorry left Kampala for Kigali at a speed of 30 km/h. If the distance between Kigali and Kampala is 280 km,

   a) After how long did the two vehicles meet?

   b) At what time did they meet?

   c) Find the distance each had covered by the time they met.

Kigali •————————— **D = 280 km** —————————• **Kampala**

Car: * DT = 4:00 am　　　　　　　　　　　　**Lorry: * DT = 4:00 am**

     S = 40 km/h　　　　　　　　　　　　　　　　**S = 30 km/h**

$$T = \frac{D}{S_1+S_2} = \frac{280\ km}{40km/h+\ 30km/h} = \frac{280\ km}{70\ km/h} = 4\ hrs$$

Time of a car = 4 hrs

Time of a lorry = 4 hrs

a) They met after 4 hrs

b) They met at DT + T = 4:00 am + 4 hrs = 8:00 am

c) -D covered by a car = S × T = $40km/h$ × 4 hrs =160 km

   -D covered by a lorry= S × T = $30km/h$ × 4 hrs =120 km

Exercises

1) Town A and B are 180 km apart. A car starts from town A and travels at an average speed of 60 km/h. At the same time a cyclist leaves town B for town A moving at 30 km/h.
   a) How far from town M does the car meet the cyclist?
   b) What time does the car take before it meets the cyclist?

2) Two motorists stared moving at the same time towards each other. Their speeds were 30 km/h and 40 km/h respectively. If the distance between them was 210 km,
   a) After how long did they meet?
   b) Find the distance each covered.

3) Sarah left Huye for Rubavu at 7:40 am moving at 60 km/h. At the same time Ana left Rubavu for Huye travelling at 50 km/h. If Huye is 220 km far from Rubavu,
   a) When did they meet?
   b) How far from Rubavu did they meet?

4) Peter and James started moving towards each other at 3:50 pm. Peter was travelling at 40 km/h while James was moving at 60 km/h. If the distance between them was 250 km,
   a) At what time did they meet?
   b) Find the distance each covered.

5) A taxi left Kampala for Bujumbura at 10:30 pm at a speed of 60 km/h. At the same time a bus left Bujumbura for Kampala travelling at 80 km/h. If Kampala is 525 km far from Bujumbura,
   a) After how long did they meet?
   b) When did they meet?
   c) At what distance from Kampala did they meet?

6) A bus left Kigali for Muhanga at 6:00 pm moving at 30km/h. At the same time a car left Muhanga for Kigali at a speed of 40km/h. If the distance between Kigali and Muhanga is 210 km,
   a) When did the two vehicles meet?
   b) Find the distance each vehicle covered.

7) Tom and Silas started the journey towards each other; moving at 30 km/h and 24 km/h respectively. If between their cities there are 135 km, at what time did they meet?

8) Town A and B are 245 km far apart. A car leaves town A for town B at 3:45 am moving at 45 km/h. At the same time a bus leaves town B for town A at a speed of 60km/h.
   a) When did they meet?
   b) How far from town A did they meet?

Finding the distance between them

Betty left town N for town m moving at 30 km/h. At the same time Sarah left town m for town N travelling at 40 km/h. If they met after 3 hours, how far is town N from town M?

Town N _____ **D = ?** _____ **Town M**

Betty: S = 30 km/h	Sarah: S = 40 km/h
T = 3 hrs	T = 3hrs
D = S × T = 30 km/h × 3 hrs	D = S × T = 40 km/h × 3 hrs
= 90 km	= 120 km

Total D = D1 + D2 = 90 km + 120 km = 210 km
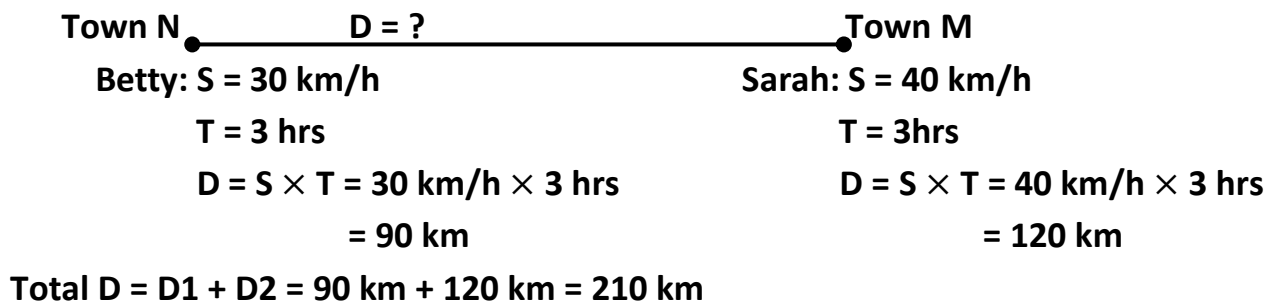
Exercises

1) Mugenzi and Murengezi left their residences and moved towards each other at 9:00 a.m. They met at 11;00 a.m. Their speeds were 40 km/h and 35 km/h respectively. What was the distance between them?

2) A car leaves Kinshasa for Kigali at 6:00 p.m at a speed of 50 km/h and at the same time a bus leaves Kigali for Kinshasa travelling at 60 km/h. If they meet at 9:30 p.m, find the distance between Kinshasa and Kigali.

3) Robert and David started moving towards each other at the same time and met after 4 hours. If their speeds were 20 km/h and 25 km/h respectively, what was the distance between them?

4) A taxi left Musanze for Kigali at 2:00 am moving at 75 km/h. At the same time a lorry left Kigali for Musanze travelling at 45 km/h. They met after $2\frac{1}{3}$ hours. How far is Kigali from Musanze?

5) Two motorists P and Q started moving at the same time towards each other. Motorist P started moving from point A and was moving at 60 km/h and motorist Q was moving from point B at 40 km/h. If they met after 3 hrs,

a) What distance did each motorist cover?

b) What is the distance between town A and town B?

6) Mico and Mary started moving towards each other at 9:40 a.m and met at 11:40 a.m. If their speeds were 40 km/h and 35 km/h respectively, what was the distance between them?

Finding one's speed

Clementine and Justine were at a distance of 60 km apart. They left their homes at 7:00 p.m and met at 9:00 p.m. If Justin moved at a speed of 12 km/h, what was the speed of Clementine?

Clementine   **D = 60 km**   **Justine**

   S = ?   **MT = 9:00 p.m**   **S = 12 km/h**

   DT = 7:00 p.m   **DT = 7:00 p.m**

   T = MT – DT = 9:00 p.m – 7:00 p.m = 2 h   **T = 2 hrs**

   D = 60 km – 24 km = 36 km   **D = S x T = 12 km/h x 2h 24km**

Speed of Clementine = $\frac{D}{T} = \frac{36\ km}{2\ h}$ = 18 km/h

Exercises

1) Town A and Town B are 240 km apart. Ana started travelling from Town A at the same time, Chantal moved from Town B. Chantal was moving at a speed of 50 km/hr. They met after 3 hours. Find Ana's speed.

2) A car left Huye for Rubavu at 3:00 a.m travelling at 40 km/h. At the same time a bus left Rubavu for Huye and they met at 5:30 a.m. If Huye is 175 km far from Rubavu, find the speed of the bus.

3) Daniel and Felix started moving towards each other at the same time and met after 2 hours. If the distance between them was 140 km and Daniel's speed was 30 km/h, what was Felix' speed?

4) Agape and James were 330 km apart. They started moving towards each other at 4:00 a.m and met at 7:00 p.m. If Agape was moving at 50 km/h, find the speed of James.

5) Town P and Q are 40 km apart. Zinda started travelling from town P and at the same time Kayitaba started moving from town Q. Zinda was moving at a speed of 80 km/h and met Kayitaba after 2 hours. At what speed was Kayitaba moving?

6) Kalisa and Mukamusoni are 240 km apart. At 7:00 a.m they started moving towards each other. Kalisa was moving at 70 km/h. If they met at 9:00 a.m,

a) What distance did Kalisa cover?

b) What was Mukamusoni's average speed?

7) Town A and B are 480 km apart. Nkubito started travelling from town A and at the same time time Nsenga started moving from town B. Nkubito was moving at a speed of 90 km/h and met Nsenga after 2 hours. At what speed was Nsenga moving?

9) <u>Moving bodies following each other</u>

A car travelling at 40 km/h left town A at 8:00 am. After 1 hour a bus travelling at 60 km/h followed it.

    a) When did the bus catch up with the car?

    b) Find the distance each covered.

    Town A

      Car: - DT = 8:00 pm

            - S = 40 km/h

            - Time of advance = 1 h

            - D of advance = S x T = 40 km/h x 1 h = 40 km

     Bus: - DT = 8:00 pm + 1h = 9:00 pm

            - S = 60 km/h

$$\textbf{T} = \frac{D}{S_2 - S_1} = \frac{40\ km}{60km/h - 40km/h} = \frac{40\ km}{20\ km/h} = \boldsymbol{2\ hrs}$$

Time of a ca = 2 hrs + 1 h = 3 hrs

Time of a bus = 2 hrs

a) At DT + T = 8:00 am + 3 hrs = 11:00 am

b) D covered by a car = S x T = 40 km/h x 3 hrs = 120 km

    D covered bu a bus = S x T = 60 km/h x 2 hrs = 120 km

<u>Exercises</u>

1) A bus leaves Kigali at 8:00 am at tge speed of 50 km/h. The same day a taxi leaves Kigali at 8:30 am and follows the bus at the speed of 80 km/h.

  a) At what time does the taxi overtake the bus?

  b) At what distance from Kigali is the bus overtaken?

2) A bus moving at a speed of 60 km/h followed a taxi which had departed 2 hours earlier. The taxi was moving at 45 km/h. When did the bus overtake the taxi if the taxi left at 9:00 am?

3) A car of speed 80 km/h started moving at 8:00 p.m. At 10:00 p.m another car travelling at 120 km/h followed from the same place.
   a) When did the second car catch up with the first one?
   b) What distance had they covered?

4) A cyclist started moving at 8 o'clock with a speed of 10 km/h. A motorcyclist started moving at 10 o'clock and followed the cyclist at the speed of 40 km/h.
   a) At what time did the motorcyclist join the cyclist?
   b) What distance did each cover?

Finding one's speed

A bus left at 3:00 am travelling at 40 km/hr. At 4:00 am a car followed it and overtook it after 4 hours. Find the speed of the car.

Bus: -DT = 3:00 am

-S = 40 km/h

- T of advance = 1h

-T = 4 hrs + 1h = 5 h

-D = S x T = 40 km/h x 5 hrs = 200 km

Car: -DT = 4:00 am

-T = 4 hrs

-D = D of bus = 200 km

-S of a car =$\frac{D}{T} = \frac{200\ km}{4\ hrs} = 50\ km/h$

Exercises

1) It took 6 hours for a motorcyclist travelling at 60 km/h to overtake a bus which had departed 2 hours earlier. If the bus started moving at 9:00 am, find the speed of the bus.

2) A ship of speed 50 km/h left the port of Mombasa at 8:10 am. At 8:40 am another ship followed. If the second ship overtook the first after 2 hours 30 minutes, find the speed of the second ship.

3) Peter left Gicumbi for Kigali moving at 42 km/h. 1 hour later Sarah followed and took him after 2 hours. At what speed was Sarah moving?

4) A taxi left city A at 4:00 a.m travelling 74 km/h. At 6:00 am a car followed and caught up with the taxi at 8:00 am. Find the speed of the car.

10. <u>Moving bodies on the graph.</u>

1) The graph below illustrates Bageni's journey from Kigali to Kayonza. Use it to answer the questions that follow.
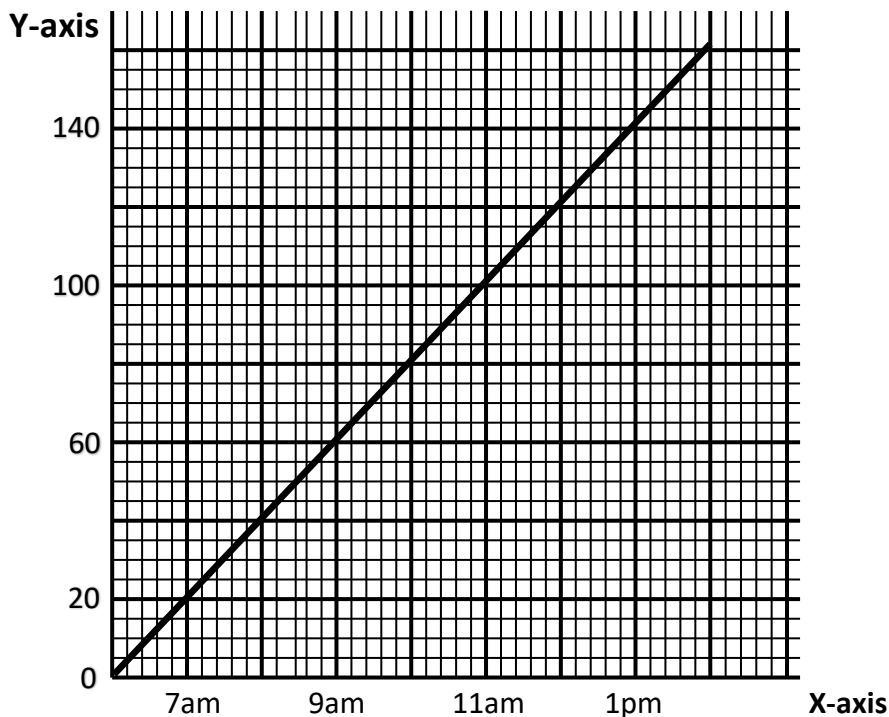
a) Find the scale on X-axis and Y-axis.

b) What distance had Bageni covered by 8:00 am?

c) By what time had Bageni covered 60km?

d) When did Bageni reach Kayonza?

e) How far is it from Kigali t Kayonza?

f) Find the distance moved in 3 hours.

g) Find the time taken to move 80 km.

h) Find Bageni's average speed.

Solution

a) Scale on X-axis: 5sq $\longrightarrow$ **1 sp** $\frac{1\ hour}{5} = \frac{60\ min}{5} = 12min$

Scale on Y-axis: 5sq $\longrightarrow$ **1 sp** $\frac{20km}{5} = \frac{20\ km}{5} = 4\ km$

b) By 8:00 am Bageni covered 40 km.

c) She covered 60 km by 9am

d) Bageni reached Kayonza at 2 pm.

e) Kigali is 160 km far from Kayonza.

f) D = 15 sq x 12 min = 180 km

g) She covered 80 km by 10:00 am

h) Average speed = $\frac{Total\ D}{Total\ T} = \frac{160\ km}{8\ hrs} = 20km/h$

2) The graph below represents Anita's journey from Musanze to Rubavu.

   a) For how long was Anita in Rubavu?

   b) By what time did Anita cover 40 km?

   c) What distance had Anita moved by 8:30 am?

   d) By what time had Anita covered 50 km in the way to Rubavu?

   e) When did Anita arrive at Rubavu?

   f) What was her average speed for the returning journey?

   g) What was her average speed for the whole journey?

3)